NOV 26 1928

Volume 17 Number 5

Journal of the Optical Society of America and Review of Scientific Instruments

EDITOR-IN-CHIEF: PAUL D. FOOTE
MELLON INSTITUTE OF INDUSTRIAL RESEARCH, UNIVERSITY OF PITTSBURGH, PITTSBURGH, PA.
ASSISTANT EDITOR-IN-CHIEF AND BUSINESS MANAGER: F. K. RICHTMYER
CORNELL UNIVERSITY, ITHACA, N. Y.
CONSULTING BUSINESS MANAGER: M. E. LEEDS, PHILADELPHIA, PA.

Associate Editors:

Term Ending Dec. 31, 1929	Term Ending Dec. 31, 1931	Term Ending Dec. 31, 1933
E. C. CRITTENDEN	FRANK ALLEN	SAUL DUSHMAN
HERBERT E. IVES	K. T. COMPTON	K. S. GIBSON
R. A. MILLIKAN	K. K. DARROW	L. O. GRONDAHL
L. P. SIEG	G. S. FULCHER	P. E. KLOPSTEG
J. P. C. SOUTHALL	I. C. GARDNER	W. B. RAYTON
H. S. UHLER	L. A. JONES	H. B. WILLIAMS

NOVEMBER, 1928

Proportion of Energy Radiated by Incandescent Solids in Various Spectral Regions – – – – – – – – – – – – – – – – L. L. HOLLADAY 329
Variation with State of the Optical Properties of Potassium and Caesium – J. B. NATHANSON 343
The Coloration of Kunzite and Hiddenite by X-Rays – – – P. L. BAYLEY 350

INSTRUMENT SECTION

Award of Prize for 1927 – – – – – – – – – – – – – – – 357
A Mounting for the Plane Grating – – – – – – – GEORGE S. MONK 358
The Use of Mirrors as "Equivalent Slits" for X-Rays – – – H. E. STAUSS 365
Locus of Secondary Image for Two Plane Mirrors Variable Independently about Parallel Axes Situated in the Respective Planes – – – – – L. E. DODD 369
A Useful Design of Tube-Voltmeter – – W. F. POWERS AND G. W. ALDERMAN 379
A Device for Accurate Timing – – – – – – – HERRICK L. JOHNSTON 381
Rotational Inertia Apparatus with Spark Recording – – HERMAN E. SEEMAN 386
Review – 388

Published Monthly by the
OPTICAL SOCIETY OF AMERICA
OFFICE OF PUBLICATION
GEORGE BANTA PUBLISHING COMPANY
450-454 Ahnaip Street
MENASHA, WISCONSIN

Entered as second-class matter May 31, 1921, at the post office at Menasha, Wis., under the Act of March 3, 1879. Acceptance for mailing at special rate of postage provided for in the Act of February 28, 1925, embodied in paragraph 4, section 412, P. L. and R. authorized June 19, 1925.

Journal
of the
Optical Society of America
and
Review of Scientific Instruments

Published monthly by the Optical Society of America in co-operation with the Association of Scientific Apparatus Makers of the United States of America. The Section on Optics is devoted to original papers on experimental, theoretical, and applied optics. The Section on Scientific Instruments is devoted to original papers on instruments of all kinds (electrical, mechanical, thermal, etc., as well as optical) for research, instruction, and industrial processes. Manuscripts for publication, are invited, whether authors are members of the Optical Society or not.

Annual subscription $5.00 dating from January of current year. Single copies $0.60. Back numbers: Vols. 1-5 (The Journal of the Optical Society, 1917-1921 inclusive) $16.00. Vol. 6-7, $5.00 per volume; Vol. 8—, $2.50 per volume. Address publishers, "George Banta Publishing Company, 450-454 Ahnaip Street, Menasha, Wisconsin, or F. K. Richtmyer, Manager, Cornell University, Ithaca, New York," for all correspondence regarding subscriptions, back numbers, changes of address, advertising rates and accounts, and missing numbers.

THE OPTICAL SOCIETY OF AMERICA

President—I. G. PRIEST, Bureau of Standards, Washington, D.C.

Vice-President—L. A. JONES, Eastman Kodak Company, Rochester, New York.

Secretary—C. C. BIDWELL, Lehigh University, Bethlehem, Pa.

Treasurer—ADOLPH LOMB, Rochester, New York.

The Council consists of the above officers, the Editor-in-Chief and the Assistant Editor-in-Chief and Business Manager of the JOURNAL, the past President, W. E. Forsythe, Nela Research Laboratories, and the following elected members: L. R. Ingersoll, University of Wisconsin; P. E. Klopsteg, Central Scientific Co.; W. F. Meggers, Bureau of Standards; A. H. Pfund, Johns Hopkins University.

The object of the Optical Society of America is to serve the interests of all who are engaged in any branch of optics from fundamental research to the manufacture of optical goods.

The Constitution provides that any one who has contributed materially to the advancement of optics shall be eligible to regular membership in the Society with the privilege of voting and holding office. Any individual or corporation interested in optics is eligible to associate membership. Annual dues including subscription to the JOURNAL are five dollars for individual associate membership, seven dollars and fifty cents for individual regular membership and fifty dollars for corporation membership. Further information may be obtained from the secretary.

Rochester, N.Y., Section of the Optical Society of America.

President—F. ALTMAN
Vice-President—S. STARK
Secretary—G. CHAMBERS
Treasurer—D. LYMAN
Councilors: L. V. FOSTER, W. W. GRAEPER, L. A. JONES, R. B. WILSEY

Journal
of the
Optical Society of America
and
Review of Scientific Instruments

Vol. 17 NOVEMBER, 1928 Number 5

PROPORTION OF ENERGY RADIATED BY INCANDESCENT SOLIDS IN VARIOUS SPECTRAL REGIONS

By L. L. Holladay

It is sometimes necessary to know the proportion of the total energy radiated by an incandescent radiator that is emitted within a given spectral region. For example, it may be desirable to know what proportion of the total energy radiated by a tungsten, carbon or black body radiator is emitted in the visible spectrum.

To make these determinations has in the past required an expenditure of much time and labor. Even in the case where the radiator is an incandescent black body, whose laws of radiation are well known, the determination has proven tedious and sometimes uncertain. To be able to make the necessary computations more easily and expeditiously is the purpose of this paper.

It will be shown that from a table or plot of the proportion of spectral energy emitted from a black body radiator at a given temperature between wave lengths zero and λ, the proportion of energy ϕ_1 emitted in a given spectral region may be computed in a moment for a black body radiator of any other temperature. Similarly, the energy emitted from the same spectral region by a non-black body radiator having color temperature T_c may be computed by multiplying the value of ϕ_1 for a black body radiator at temperature T_c, by a simple factor G.

BLACK BODY RADIATORS

The distribution of energy throughout the spectrum of a black body radiator is according to Planck's formula:

$$E_\lambda = \frac{C_1}{\lambda^5} \frac{1}{e^{C_2/\lambda T} - 1} \qquad (1)$$

where $E_\lambda\, d\lambda$ is the amount of energy contained in a $d\lambda$ length of spectrum at wavelength λ, that is emitted per unit of time per unit of area by a black body at the absolute temperature T; C_1 and C_2 are constants; and e is the base of the natural system of logarithms.

The total energy emitted per second per unit of area of a black body is

$$E=\int_0^\infty E_\lambda d\lambda=\int_0^\infty \frac{C_1}{\lambda^5}\frac{d\lambda}{e^{C_2/\lambda T}-1} \tag{2}$$

$$=\frac{6C_1\alpha}{C_2^4}T^4=\sigma T^4 \tag{3}$$

where α is a constant $=1.0823$ approximately and σ is the Stefan-Boltzmann constant, having a value of 5.709×10^{-12} watts per sq. cm. $\times\text{deg}^{-4}$.

The energy in the region of the spectrum from wave lengths O to λ is

$$E_1=\int_0^\lambda E_\lambda d\lambda \tag{4}$$

or when expressed as a ratio of the total energy in the spectrum is from equations 2 and 4 equal to

$$\phi=\frac{E_1}{E}=\frac{1}{E}\int_0^\lambda E_\lambda d\lambda \tag{5}$$

$$=\frac{C_2^4}{6\alpha}\int_0^{\lambda T}\frac{d(\lambda T)}{(\lambda T)^5(e^{C_2/\lambda T}-1)} \tag{6}$$

As far as the author is aware, the value of the ratio ϕ has hitherto been computed by determining the values of E_1 and E independently for each temperature, the value of E_1 generally being obtained by the step by step method for the particular temperature of the radiator, and the value of E being computed by the Stefan-Boltzmann law. However, by a new method now to be presented, the integration may be performed once and for all and the results used for a black body radiator of any temperature.

This integration may be accomplished for any temperature, by combining Planck's equation with Wien's displacement law connecting temperature T and wave length λ_m at which the maximum energy occurs; to-wit

$$\lambda_m T=A\text{, a constant} \tag{7}$$

Substituting this value of λ_m in Planck's equation (Eqt. No. 1), we obtain the following expression for the maximum intensity of spectral energy:

$$E_m = \frac{C_1\left(\frac{A}{T}\right)^{-5}}{e^{C_2/A}-1} \tag{8}$$

Dividing E_λ the intensity of spectral energy at wave length λ by E_m, the maximum intensity, we obtain from equations 1 and 8:

$$\frac{E_\lambda}{E_m} = \left(\frac{A}{\lambda T}\right)^5 \frac{e^{C_2/A}-1}{e^{C_2/\lambda T}-1} \tag{9}$$

This ratio has been computed by Frehafer and Snow of the Bureau of Standards for a wide range of values λT and is reproduced in part in Table 1 and is shown graphically in curve A, Fig. 1.

By multiplying each side of equation 9 through by $d(\lambda T)$ and integrating it from zero to infinity, we obtain

$$\int_0^\infty \frac{E_\lambda}{E_m} d(\lambda T) = \frac{6\alpha A^5}{C_2^4}(e^{C_2/A}-1) \tag{10}$$

Similarly if each side of equation 9 be multiplied by $d(\lambda T)$ and the integration be indicated for the limits zero and λT and then the left and right hand sides of the resultant equation be respectively divided by the left and right sides of equation 10 we obtain

$$\phi = \frac{\int_0^{\lambda T} \frac{E_\lambda}{E_m} d(\lambda T)}{\int_0^\infty \frac{E_\lambda}{E_m} d(\lambda T)} = \frac{C_2^4}{6\alpha} \int_0^{\lambda T} \frac{d(\lambda T)}{(\lambda T)^5 (e^{C_2/\lambda T}-1)} \tag{11}$$

This value ϕ is numerically equal to ϕ, the expression given in equation 6 for the proportion of black body radiant energy in the spectral region between wave lengths zero and λ.

With $\alpha = 1.0823$, $A = 2890$ micron degrees, $C_2 = 14350$ micron degrees, and $e^{\frac{C_2}{A}} - 1 = 142.37$, we find from equation 10 that

$$\int_0^\infty \frac{E_\lambda}{E_m} d(\lambda T) = 4395.4 \text{ micron degrees} \tag{12}$$

Therefore we may practically determine the proportion of spectral energy in any region of the spectrum of a black body radiator by

TABLE 1. *Showing energy distribution of a black body radiator. In the third column is tabulated the proportion of energy emitted by a black body radiator at temperature T in the spectral region between wave lengths zero and λ microns.*

Product of wave length λ in microns times absolute temperature in degrees Kelvin. λT in micron degrees.	Spectral intensity of radiation in terms of maximum, according to Frehafer and Snow* E_λ/E_m (See equation 9)	Proportion of spectral energy in region from ultraviolet limit to λT ϕ (See equation 13)
400	7.367×10^{-10}	
420	3.185×10^{-9}	8.013×10^{-12}
440	1.193×10^{-8}	3.947×10^{-11}
460	3.943×10^{-8}	1.480×10^{-10}
480	1.171×10^{-7}	4.828×10^{-10}
500	3.154×10^{-7}	1.417×10^{-9}
520	7.818×10^{-7}	3.805×10^{-9}
540	1.799×10^{-6}	9.466×10^{-9}
560	3.874×10^{-6}	2.197×10^{-8}
580	7.869×10^{-6}	4.797×10^{-8}
600	1.514×10^{-5}	9.907×10^{-8}
620	2.781×10^{-5}	1.949×10^{-7}
640	4.889×10^{-5}	3.664×10^{-7}
660	8.275×10^{-5}	6.614×10^{-7}
680	1.351×10^{-4}	1.150×10^{-6}
700	2.135×10^{-4}	1.935×10^{-6}
720	3.276×10^{-4}	3.155×10^{-6}
740	4.897×10^{-4}	5.014×10^{-6}
760	7.136×10^{-4}	7.751×10^{-6}
780	1.018×10^{-3}	1.169×10^{-5}
800	1.419×10^{-3}	1.723×10^{-5}
840	2.615×10^{-3}	3.525×10^{-5}
880	4.503×10^{-3}	6.718×10^{-5}
900	5.785×10^{-3}	9.051×10^{-5}
920	7.326×10^{-3}	1.204×10^{-4}
960	1.134×10^{-2}	2.046×10^{-4}
1000	1.682×10^{-2}	3.318×10^{-4}
1050	2.609×10^{-2}	5.742×10^{-4}
1080	3.314×10^{-2}	7.764×10^{-4}
1120	4.441×10^{-2}	1.128×10^{-3}
1160	5.794×10^{-2}	1.593×10^{-3}
1200	7.391×10^{-2}	2.192×10^{-3}
1250	9.720×10^{-2}	3.161×10^{-3}
1300	.1243	4.421×10^{-3}
1400	.1887	7.962×10^{-3}
1500	.2647	.01311
1600	.3486	.02007
1700	.4363	.02900
1800	.5241	.03993
1900	.6086	.05282

* loc. cit.

TABLE 1. (continued)

λT	E_λ/E_m	ϕ
2000	.6871	.06757
2100	.7579	.08402
2200	.8196	.1020
2300	.8719	.1212
2400	.9147	.1416
2500	.9479	.1628
2600	.9722	.1846
2700	.9888	.2069
2800	.9974	.2296
2890	1.0000	.2500
3000	.9967	.2750
3100	.9886	.2976
3200	.9762	.3200
3300	.9603	.3420
3400	.9418	.3636
3500	.9209	.3848
3600	.8982	.4055
3700	.8742	.4257
3800	.8492	.4453
3900	.8236	.4643
4000	.7975	.4827
4200	.7453	.5178
4400	.6938	.5506
4600	.6440	.5810
4800	.5967	.6092
5000	.5521	.6353
5400	.4715	.6818
5800	.4023	.7215
6000	.3716	.7392
6400	.3177	.7705
6800	.2723	.7973
7000	.2523	.8092
7400	.2173	.8305
7600	.2019	.8401
7800	.1878	.8489
8000	.1748	.8572
8400	.1519	.8720
8800	.1324	.8850
9000		.8908
9200	.1159	.8963
9600	.1018	.9062
10000	.08970	.9149
10800	.07036	.9294
11600	.05588	.9408
12400	.04488	.9499
13200	.03644	.9573
14000	.02986	.9633

TABLE 1. (continued)

λT	E_λ/E_m	ϕ
14800	.02469	.9683
15600	.02059	.9724
16000	.01886	.9742
17000	.01525	.9781
18000	.01246	.9812
19000	.01027	.9838
20000	.008548	.9859
21000		.9877
Infinity	0	1.

summing the products $\frac{E_\lambda}{E_m}\Delta(\lambda T)$ between the limits $\lambda_1 T$ and $\lambda_2 T$ and dividing the sum by 4395; or

$$\phi_{\lambda_2 T} - \phi_{\lambda_1 T} = \frac{1}{4395} \sum_{\lambda_1 T}^{\lambda_2 T} \frac{E_\lambda}{E_m} \Delta(\lambda T) \tag{13}$$

By employing this method of computation and using the ratios $\frac{E_\lambda}{E_m}$ for black body radiators given by Frehafer and Snow[1], the proportions ϕ of energy in the spectral region extending from $\lambda = 0$ to wave lengths corresponding to various values of λT have been computed and tabulated in the third column of Table 1. These values of ϕ are also shown graphically in Figs. 1 and 2.

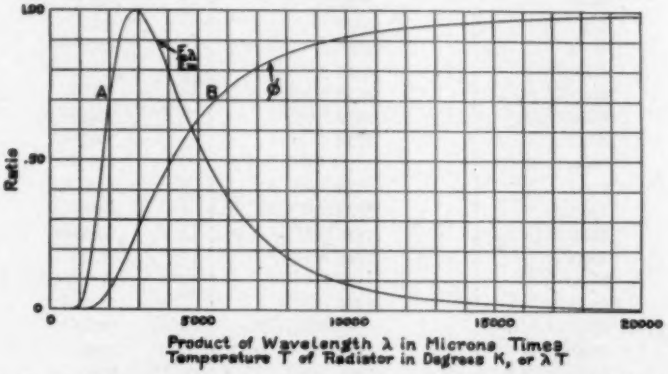

FIG. 1. *Showing the spectral distribution of energy from a black body radiator at a temperature T°K. Curve A shows the ratio E_λ/E_m of spectral intensity of radiant energy at wave length λ to the maximum intensity, plotted as ordinate; and the product of wave length λ in microns times the temperature T in degrees Kelvin or λT, plotted as abscissa. Curve B shows the proportion of radiant energy ϕ emitted in the region between zero and λT micron degrees.*

[1] M. K. Frehafer and C. L. Snow, Miscellaneous Publication of the Bureau of Standards; No. 56, March 21, 1925.

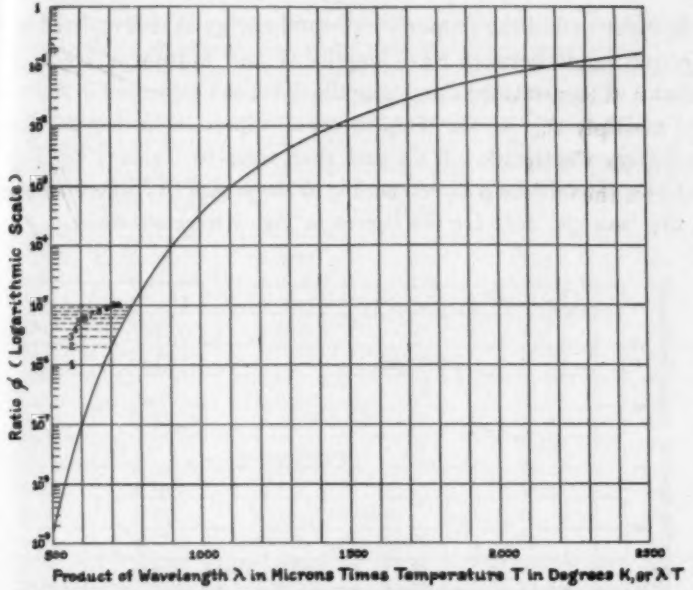

FIG. 2. *Proportion of radiant energy ϕ emitted by a black body radiator in the spectral region between zero and λT micron degrees, plotted to a logarithmic scale.*

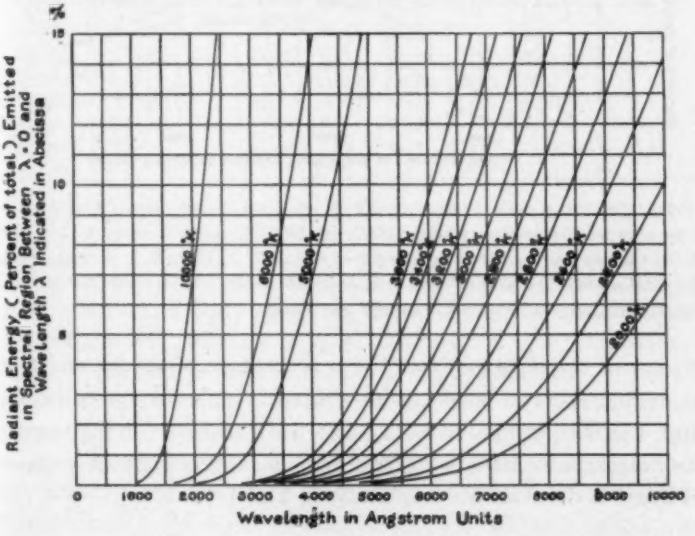

FIG. 3. *Per cent of radiant energy emitted in spectral region between wave lengths zero and λ Angstroms for a black body radiator at various temperatures.*

In order to find the proportion of total energy that is emitted in the spectral region between wave lengths O and λ from a black body radiator of temperature T, we take the value of λ expressed in microns* and multiply this by the temperature T expressed in degrees Kelvin (=degrees Centigrade+273°) and then refer to Table 1 or Figs. 1 and 2 for the value of ϕ corresponding to the product λT micron degrees. In this way the data for the curves in Fig. 3 were obtained.

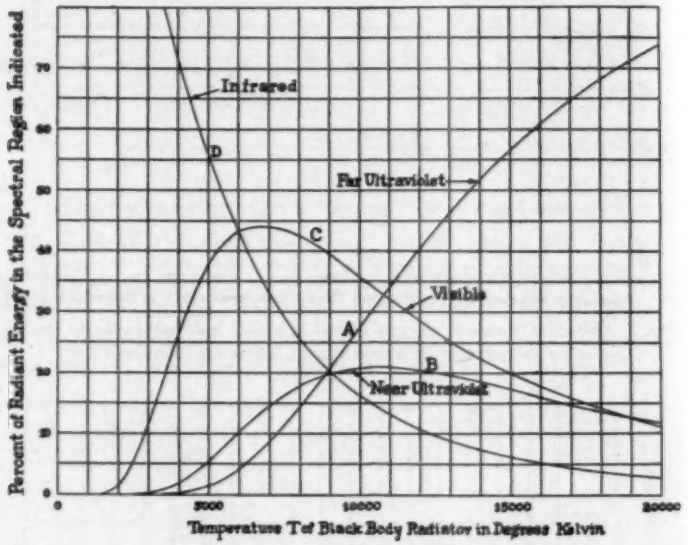

FIG. 4. *Per cent of radiant energy emitted in four given spectral regions by a black body radiator at various temperatures between 1000° and 20000°K. Curve A shows per cent emitted in the far ultraviolet region between wave lengths λ0 and λ3000 A; Curve B, in the near ultraviolet region between λ3000 and λ4000; Curve C, in the visible region, between λ4000 and λ7600, and Curve D, in the infrared region, between λ7600 and infinity.*

By use of the data in Table 1, the percentage of energy emitted in each of four spectral bands, (which together embrace the entire spectrum), has been computed for a black body radiator having temperatures ranging from 1000 to 20000 degrees K. These data are presented in Table 2 and are shown graphically in Fig. 4.

* 1 micron = 1000 millimicrons = 10000 Angstroms.

TABLE 2. *Proportions of spectral energy emitted in the far ultraviolet, near ultraviolet, visible and infrared regions of the spectrum of a black body radiator at various temperatures.*

Temperature of black body radiator, in degrees K.	Per cent of spectral energy emitted in various regions of the spectrum of a black body radiator.				
	Far U.V. or λ0 to λ3000A	Near U.V. or λ3000 to λ4000	Visible or λ4000 to λ7600	Infrared or λ7600 to infinity	Total spectrum or λ0 to infinity
1000°K	0.0000	0.0000	0.001	100.0	100
1500	0.0000	0.0000	0.115	99.9	100
1800	0.0000	0.0003	0.668	99.3	100
2000	0.0000	0.0017	1.43	98.6	100
2200	0.0001	0.0067	2.62	97.4	100
2400	0.0003	0.020	4.28	95.7	100
2500	0.0006	0.032	5.25	94.7	100
2600	0.0012	0.051	6.35	93.6	100
2700	0.0021	0.076	7.41	92.5	100
2800	0.0035	0.109	8.78	91.1	100
2900	0.0058	0.153	10.13	89.7	100
3000	0.0091	0.210	11.51	88.3	100
3100	0.0138	0.281	12.97	86.7	100
3200	0.0205	0.369	14.44	85.2	100
3300	0.0296	0.472	15.95	83.5	100
3500	0.057	0.74	19.00	80.2	100
4000	0.219	1.79	26.39	71.6	100
5000	1.31	5.45	37.77	55.5	100
6000	3.99	10.17	43.36	42.5	100
8000	14.16	17.84	42.59	25.4	100
10000	27.5	20.77	35.74	16.0	100
12000	40.5	20.37	28.49	10.6	100
15000	56.6	17.31	19.89	6.2	100
20000	73.9	11.80	11.32	2.96	100

NON-BLACK BODY RADIATORS

In the study of the spectral distribution of energy from tungsten and other non-black body radiators, the black body is usually referred to as a standard, since for a given temperature, its distribution of energy is determinate theoretically and experimentally.

When an incandescent non-black body radiator operates at temperature T and the temperature of a black body radiator with which it is compared is raised to a temperature T_c such that its color appears equal to that of the non-black body radiator, then the non-black body radiator is said to have a color temperature T_c. At these temperatures the spectral intensities of energy emitted by the non-black body

radiator at wavelengths λ_1 and λ_2 bear a definite ratio to the intensities from the black body radiator at the same wave lengths. This ratio ϵ_c is called the color emissivity of the non-black body radiator.

The spectral energy radiated in the region between wave lengths λ_1 and λ_2 by the non-black body radiator at its temperature T will therefore be ϵ_c times the energy radiated in the same spectral region by the black body radiator at temperature T_c or is

$$E' = \int_{\lambda_1}^{\lambda_2} \epsilon_c \frac{C_1}{\lambda^5} \frac{d\lambda}{\left(\frac{C_2}{e^{\lambda T_c}} - 1\right)} \tag{14}$$

$$= T_c^4 \int_{\lambda_1 T_c}^{\lambda_2 T_c} \epsilon_c \frac{C_1}{(\lambda T_c)^5} \frac{d(\lambda T_c)}{(e^{C_2/\lambda T_c} - 1)} \tag{15}$$

If the color emissivity ϵ_c for the non-black body radiator be assumed constant throughout the region between wave lengths λ_1 and λ_2 then

$$E' = \epsilon_c T_c^4 \int_{\lambda_1 T_c}^{\lambda_2 T_c} \frac{C_1}{(\lambda T_c)^5} \frac{d(\lambda T_c)}{e^{C_2/\lambda T_c} - 1} \tag{16}$$

The total energy radiated per second from a square centimeter of the non-black body radiator at temperature T is ϵ_t times the energy radiated by a black body at the same temperature T or from equation 3:

$$E_t = \epsilon_t \sigma T^4 = \epsilon_t \frac{6 C_1 \alpha}{C_2^4} T^4 \tag{17}$$

Therefore the proportion of the total energy emitted in the region between wave lengths λ_1 and λ_2 by the non-black body radiator at temperature T is from equations 16 and 17 equal to

$$\phi' = \frac{E'}{E_t} = \frac{\epsilon_c}{\epsilon_t} \left(\frac{T_c}{T}\right)^4 \frac{C_2^4}{6\alpha} \int_{\lambda_1 T_c}^{\lambda_2 T_c} \frac{d(\lambda T_c)}{(\lambda T_c)^5 (e^{C_2/\lambda T_c} - 1)} \tag{18}$$

By comparing this result for a non-black body radiator with that given in equations 6 and 11 for a black body radiator it may be seen that the proportion of the total radiant energy emitted in a given spectral region from a non-black body radiator is

$$\phi' = \frac{\epsilon_c}{\epsilon_t} \left(\frac{T_c}{T}\right)^4 (\phi_{\lambda_2 T_c} - \phi_{\lambda_1 T_c}) = G(\phi_{\lambda_2 T_c} - \phi_{\lambda_1 T_c}) \tag{19}$$

where $(\phi_{\lambda_2 T_c} - \phi_{\lambda_1 T_c})$ is the proportion emitted in the same spectral

region from a black body radiator at temperature T_c, the color temperature of the non-black body radiator; and G is the factor

$$\frac{\epsilon_c}{\epsilon_t}\left(\frac{T_c}{T}\right)^4$$

The value of factor G for the visible spectrum may be shown to be a ratio of luminous efficiencies as follows: If b be the brightness in candles per square centimeter normal to a black body radiator at temperature T_c, the total lumens emitted per square centimeter are πb. Also the lumens emitted per watt are

$$l = \frac{\pi b}{\sigma T_c^4} \tag{20}$$

Also if b_1 be the brightness and ϵ_t be the total emissivity of the non-black body radiator at temperature T, and if Lambert's cosine law holds true, its luminous efficiency is

$$l_1 = \frac{\pi b_1}{\sigma \epsilon_t T^4} \tag{21}$$

By definition the "color emissivity" ϵ_c is also the ratio of the normal brightness of the non-black body radiator at temperature T to the normal brightness b of the black body radiator at a temperature T_c which is equal to the color temperature of the non-black body radiator. Therefore

$$\epsilon_c = \frac{b_1}{b} \tag{22}$$

Combining equations 19, 20, 21 and 22, we find that

$$\phi' = \frac{l_1}{l}(\phi_{\lambda_1 T_c} - \phi_{\lambda_1 T_c}) \tag{23}$$

or

$$G = \frac{\epsilon_c}{\epsilon_t}\left(\frac{T_c}{T}\right)^4 = \frac{l_1}{l} \tag{24}$$

In Table 3 are given numerous data upon tungsten taken from publications by W. E. Forsythe and A. G. Worthing and H. E. Ives[2,3] Values of factor G for the visible spectrum have been computed for

[2] W. E. Forsythe and A. G. Worthing, The Properties of Tungsten and the Characteristics of Tungsten Lamps, The Astrophysical Journal, *61*, April, 1925.

[3] H. E. Ives, The Luminous Properties of the Black Body, J.O.S.A. & R.S.I., *12*, February, 1926.

TABLE 3. *Data on tungsten compiled from various published data by W. E. Forsythe and A. G. Worthing, together with comparative values of factor G for the visible spectrum as computed by use of either emissivity or luminous efficiency data. According to equations 19 and 23, G is the factor by which the proportion of energy in a given region of the spectrum of a black body at temperature T_c must be multiplied in order to obtain the proportion of energy emitted in the same spectral region by tungsten when of color temperature T_c.*

Temperature of tungsten filament in deg. K		Emissivities		Luminous efficiency in lumens per watt.		Computed value of factor G for the visible spectrum.	
True temp. T	Color temp. T_c	Color emis. ϵ_c	Total emis. ϵ_t	Tungsten at temp. T l_1	Black body at temp. T_c l	$\frac{\epsilon_c}{\epsilon_t}\left(\frac{T_c}{T}\right)^4$	$\frac{l_1}{l}$
1500	1517	.383	.192	0.20		2.09	
1800	1825	.376	.236	1.16	.67	1.64	1.73
2000	2033	.370	.260	2.78	1.75	1.52	1.59
2200	2242	.364	.279	5.47	3.67	1.41	1.49
2400	2452	.350	.296	9.37	6.70	1.29	1.40
2600	2663	.353	.311	14.28	10.8	1.25	1.32
2800	2878	.347	.323	20.43	16.0	1.20	1.28
3000	3094	.343	.334	27.1	22.1	1.16	1.22
3200	3311	.338	.341	34.6	28.9	1.14	1.19
3300	3422	.335	.344	38.5		1.13	
3400	3533	.332	.348	42.6		1.12	
3500	3646	.329	.351	45.9		1.10	
3655	3817	.324	.354	53.1		1.09	

TABLE 4. *Data on sundry non-black body radiators, according to E. P. Hyde, together with computed values of factor G for the visible spectrum.*

Type of radiator	Color temp. of radiator in degrees K T_c	Luminous efficiency in lumens per watt		Computed value of factor G, for the visible spectrum or $\frac{l_1}{l}$
		Non-black body radiator at temp. T l_1	Black body radiator at temp. T_c l	
Black body	1700	0.34	.34	1.
" "	2160	2.8	2.8	1.
Untreated carbon	2160	3.5	2.8	1.25
" " "	1700	.39	.34	1.15
Flashed carbon	1700	.41	.34	1.20
" "	2160	3.7	2.8	1.32
Graphitized carbon	1700	.41	.34	1.20
" "	2160	3.7	2.8	1.32
Osmium	1700	.72	.34	2.12
"	2160	4.9	2.8	1.75
Tantalum	1700	.5	.34	1.47
"	2160	3.9	2.8	1.39
Tungsten	1700	.59	.34	1.73
"	2160	4.4	2.8	1.57

tungsten by both expressions for G and are given for comparison in Table 3. In Table 4 are presented data upon several other types of non-black body radiators[4,5] These data are rather meagre, but give values of factor G which apparently are of the correct general order of magnitude. Table 5 contains normal operating data upon numerous incandescent lamps which have filaments of the non-black body type.

TABLE 5. *Data on various types of incandescent lamps and computed values of factor G for the visible spectrum and the proportion of total energy emitted by tungsten in the visible spectrum (λ4000 to λ7000).*

Lamp	Lumens emitted per watt l_1	Maximum temperature in deg. K	Average color temp. T_c in deg. K	Maximum brightness of filament candles /(cm)² b_1	Probable value of factor G for the visible spectrum G	Proportion of total radiant energy emitted in the visible spectrum	
						by a black body at temp. T_c ϕ_v	by a non-black body radiator $G\phi_v$
(Vacuum lamps)							
50-watt carbon			2065		1.25	.01776	.022
50-watt gem			2130		1.25	.02157	.027
2.5-w.p.c. gem			2195		1.25	.02587	.032
4-w.p.c. carbon			2080		1.25	.01869	.023
3.1-w.p.c. carbon			2165		1.20	.02379	.028
2-w.p.c. osmium			2185		1.72	.02513	.043
50-watt tantalum			2300		1.23	.03388	.042
2-w.p.c. tantalum			2260		1.24	.03062	.038
1.25-w.p.c. tungsten			2400		1.33	.04265	.057
10-watt tungsten	8.		2410		1.32	.04363	.058
25-watt "	9.8		2450		1.31	.04743	.062
40-watt "	10.0		2460		1.30	.04747	.062
60-watt "	10.1		2465		1.30	.04885	.063
(Regular gas-filled tungsten lamps)							
50-watt tungsten	10.0	2685	2670	469	1.25	.07165	.090
75-watt	11.8	2735	2705	563	1.24	.07592	.094
100-watt	12.9	2760	2740	605	1.23	.08020	.099
200-watt	15.2	2840	2810	781	1.22	.08897	.109
300-watt	16.3	2870	2840	862	1.21	.09314	.113
500-watt	18.1	2930	2920	1015	1.20	.1003	.120
1000-watt	20.0	2990	2980	1225	1.19	.1122	.133
2000-watt	21.2	3020	3000	1350	1.18	.1151	.136
(Special tungsten lamps.)							
1000-watt stereopticon	24.2	3185	3175	2065	1.16	.1407	.163
900-watt movie	27.3	3290	3220	2660	1.15	.1476	.170
10-KW.	31.0	3350	3300	3050	1.14	.1595	.182
30-KW.	31.0	3350	3300	3050	1.14	.1595	.182

[4] A. G. Worthing, Emissive Powers and Temperatures of Non-black Bodies, Bulletin Am. Inst. Mining and Metal. Eng., No. 153, Sept. 1919.

[5] W. E. Forsythe, Color Match and Spectral Distribution, J.O.S.A. & R.S.I., 7, Dec. 1923.

When the spectral energy curve of tungsten at a certain temperature is compared with that of a black body at the color temperature of the tungsten, there is the same spectral distribution throughout the visible region and probably throughout the ultraviolet region, and perhaps in the near infrared region, but not in the remainder of the infrared region. For this reason, factor G is satisfactory for determining the fractional portion of the energy in the visible and probably the ultraviolet portions of the spectrum, but not for the infrared portion. However, the portion in the infrared region may be determined by subtracting the portion in the ultraviolet and visible regions from unity.

Thanks are hereby expressed to M. Luckiesh, W. E. Forsythe, and A. G. Worthing for helpful criticisms in preparing this paper.

LIGHTING RESEARCH LABORATORY,
NATIONAL LAMP WORKS OF GENERAL ELECTRIC COMPANY,
NELA PARK, CLEVELAND, OHIO.

VARIATION WITH STATE OF THE OPTICAL PROPERTIES OF POTASSIUM AND CAESIUM

By J. B. Nathanson

ABSTRACT

The optical constants of potassium and caesium were determined for the solid and liquid states. Observations were made at temperatures above and below the melting points of the metals. The glass cell containing the alkali mirror was heated by means of a small electrical oven enclosing the mirror and containing suitable apertures for the incident and reflected light rays. The method of observation was that employed in previous investigations.

The optical constants were computed by means of Drude's equations. When the metal changed from the solid to the liquid state, it was found that the angle of azimuth of restored plane polarization, as well as the phase difference of the components of the light vector parallel and perpendicular to the plane of incidence, changed by only a small amount. After due allowance for the effects of strains in the glass cells, it was concluded that the optical properties of the liquid alkali surface are closely identical with those of the solid alkali surface.

The optical constants for a number of metals and their alloys have been determined for both the solid and liquid states. Various investigators have stressed the difficulty of obtaining an optical surface for a solid metal, which shall be characteristic of the metal itself, rather than of the manner of grinding and polishing the surface. Accordingly, the results obtained for the same metal by different investigators have not been in very good agreement with each other. Barring surface contamination, greater agreement should be expected from the study of metals in the liquid state, since good smooth surfaces can be obtained without the aid of any polishing process.

A study of the literature reveals the fact that only for comparatively few metallic elements, have the optical constants been determined for both the solid and liquid states. The metals about which we have such information are bismuth, lead, cadmium, tin, mercury, and potassium. A study of the data shows that the per cent difference in the values of the refractive indices for the two states of any metal, is greater than the corresponding per cent difference in the values of the coefficients of absorption. Unfortunately there are even considerable differences between the values obtained by different investigators for the same metal in the same state. It is interesting to note that in but few cases have observations been made on the identical mirror in the solid and liquid states. It is therefore not surprising to find that the optical constants as given for a metal are different for the two states. Hence it

becomes difficult to conclude whether these differences in the optical constants are actually due to a change in state, or are merely inherent in the nature of the experimental work which is carried out by different observers on different mirrors of the same metal.

Kent[1] has determined the optical constants of bismuth, cadmium, tin, lead, and their binary alloys, all of these metals being in the liquid state. A comparison of Kent's results with those obtained by other investigators for the same metals but in the solid state, reveals a marked difference in the optical constants for these metals for the two states. From this difference, Kent concluded that for the liquid state, the electrical resistivities of the metals agreed satsifactorily with the values as computed from the optical constants by means of Drude's theory of metallic dispersion. However for the metal in the solid state, no such saitsfactory agreement could be obtained.

On the other hand, Morgan's[2] results for potassium indicate comparatively small changes in the optical constants, the refractive indices being 0.062 and 0.057 for the solid and liquid states respectively, while the corresponding values of the coefficients of absorption are 24.4[3] and 23.6. Morgan used the same mirror to obtain the data for the two states.

In one other case have the optical constants been determined for both states and for the same mirror. Haak and Sissingh[4] working with a mirror of mercury, found no abrupt change in the phase difference of the two components of the reflected light as the temperature of the mercury was reduced to −80°C. They came to the conclusion therefore, that optically speaking, solid and liquid mercury are identical.

The optical constants of the alkali metals have been determined by a number of investigators.[5] The metals by virtue of their relatively low melting points, lend themselves quite suitably to the study of their optical properties in the solid and liquid states. In the present investigation, the metals studied were potassium and caesium. These were contained in narrow cylindrical glass cells whose ends were closed by windows of optical glass. The metal being in intimate contact with the glass window, suffered no change in the smoothness of its surface with change of state. It was felt that a truer comparison of the optical constants for the two states could thus be obtained.

[1] Phys. Rev., *14*, p. 459; 1919.
[2] Phys. Rev., *20*, p. 203; 1922.
[3] Morgan has given the value of 19.3 for the coefficient of absorption of potassium in the solid state, but a recomputation of his results yields the value 24.4.
[4] K. Akad. Amsterdam Proc., *21*, p. 691; 1919.

EXPERIMENTAL METHOD

The method used to determine the optical constants was the same as that[5] previously employed by the writer in the study of the optical properties of rubidium and caesium in the solid state. Briefly, the method consists in allowing monochromatic light polarized in a plane making an angle of 45° with the plane of incidence, to be reflected from the mirror at an angle of 45°. The phase difference of the vertical and horizontal components of the reflected elliptically polarized light is then determined by means of a Babinet compensator, while the azimuth of restored plane polarization is determined by means of the analyzing nicol prism in the eyepiece. As in the previous work the alkali metal was contained in a small glass cell, to which were cemented two

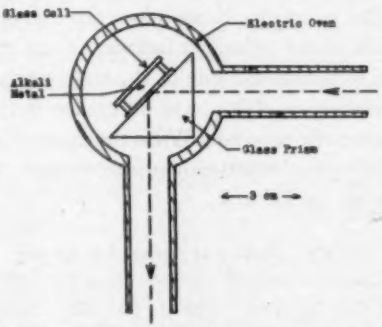

FIG. 1. *Mounting of the alkali mirror on the spectrometer.*

plane plates of crown glass. These plates were examined for any strain before and after cementing to the walls of the cell. One of the windows of the cell was placed in optical contact by means of a little cedar oil, with the hypothenuse side of a right angle prism of crown glass, as shown in Fig. 1. Light entering normally one leg of the prism was reflected at the metal-glass boundary, and emerged normally from the other leg of the prism.

The caesium mirrors were made by first producing the metal in a vacuum, from a heated mixture of caesium chloride and calcium, and after several distillations, filling the glass cell with the purified metal. Two methods were employed to produce the potassium mirrors. One method was to form the potassium mirror in the same manner as the caesium mirror, except that the start was made with metallic potassium of Kahlbaum purity. The other method used was one similar to that

employed by the Duncans,[5] in which the potassium after distillation and while still as a liquid is pressed into the heated cell by means of dried air admitted to the previously evacuated apparatus.

In order that observations could be made on the alkali metal in the liquid as well as in the solid state while in place on the spectrometer, the glass prism and mirror were enclosed in a small electrically heated oven, Fig. 1, having tubular openings for the incident and emergent light rays. Except for three contact points, the oven containing prism and mirror was thermally insulated from the remainder of the spectrometer. The oven was wound to insure uniform heating, and the temperature of the interior could be maintained quite constant for several hours. The manner of heating the mirror is important in order to avoid spurious effects due to strains developed in the glass parts with change in temperature. For example, if one observes the phase difference of the two components of the polarized light after suffering total internal reflection from the hypothenuse side of the right angle prism, one finds a considerable variation in this phase difference if the heating of the prism be made purposely uneven. With the type of heater used however, the phase difference remained quite constant with variation of temperature as shown in Table 1.

TABLE 1. *Position of Babinet Compensator.*

Temp.°C	25	26	29	35	36.5	50
Position	28.068	28.063	28.062	28.065	28.069	28.066

The values given are the average of a number of observations at the temperatures indicated. The maximum variation in the position of the compensator represents a phase change of about 16 minutes of arc. It is therefore believed that with the type of heater used, the strains developed in the glass parts were reduced to a minimum.

The mode of procedure in making observations on the alkali mirrors, was first to observe the phase difference and azimuthal angle for the metal in the solid state at room temperature. The mirror was then

[5] P. Drude, Ann. der Phys., *64*, p. 159; 1898. R. W. and R. C. Duncan, Phys. Rev., *36*, p. 294; 1913. B. Meese, Gott. Nachr., p. 530; 1913. J. B. Nathanson, Astrophys. J., *44*, p. 137; 1916; Phys. Rev. *11*, p. 227; 1918; Phys. Rev., *25*, p. 75, 1925. Mabel K. Frehafer, Phys. Rev., *15*, p. 110; 1920. R. Morgan, Phys. Rev., *20*, p. 203; 1922. H. E. Ives and A. L. Johnsrud, J.O.S.A. & R.S.I., *15*, p. 374; 1927.

heated slowly to the temperature desired, at which the mirror was held for several hours before proceeding with the observations. Temperatures were chosen a few degrees below and above the melting points of the metals when making observations on the effect of a change in state.

RESULTS

The results for the best mirrors of potassium and caesium are given in Tables 2 and 3. The values of the index of refraction n, the coefficient of absorption k, and the reflecting power R for normal incidence, have been computed by means of Drude's equations from the observed values of the phase difference Δ, and twice the azimuthal angle of restored plane polarization 2ψ. The observed angles are for the metal-glass surface, but the computed values of the optical constants are for the metal-air surface. The index of refraction for the metal-air boundary was obtained from that observed for the metal-glass surface by multiplying the latter by the refractive index of the glass window of the cell.

TABLE 2. *Potassium* (*m.p.* = 62.3°C)
Wave length = 5890A

Mirror No.	Temp. °C	State	Δ	2Ψ	Change in Δ	Change in 2Ψ	n	k	R %
9	23.6	solid	126°25′	88°37′	−2°11′	− 6′	0.073	24.8	93.4
9	53.3	solid	124°14′	88°31′	−0°40′	−19′	0.075	22.9	92.6
9	71.3	liquid	123°34′	88°12′			0.089	18.9	91.1
2	21.1	solid	124°23′	88° 8′	−0°42′	−13′	0.094	18.3	91.0
2	68.0	liquid	123°41′	87°55′			0.103	16.4	89.9

Potassium mirror No. 9 was made by the Duncan method, while No. 2 was made by the method employed for making the caesium mirrors.

It would seem from an inspection of the data in Tables 2 and 3, that there are no pronounced changes in the relative phase differences or azimuthal angles when potassium or caesium change in state. The tendency seems to be for the phase difference Δ, and the azimuthal angle ψ, to diminish somewhat with increase in temperature, the changes in 2ψ being much less than the corresponding changes in Δ. The average change in 2ψ for caesium is quite inappreciable. Examination of Morgan's[2] data on potassium for a wave length of 5461A, discloses a similar experience, the phase difference dimishing from 119°30′ at a

TABLE 3. *Caesium (m.p.=26.4°C)*

Wave length	Temp. °C	State	Δ	2Ψ	Change in Δ	Change in 2Ψ	n	k	R %
5400A	23.6	solid	111°51′	82°16′	−1°38′	−6′	0.308	3.74	59.5
5400	32.9	liquid	110°13′	82°10′			0.307	3.55	57.6
5890	23.7	solid	115°47′	82° 8′	−1°32′	+7′	0.329	3.97	62.0
5890	33.7	liquid	114°15′	82°15′			0.318	3.92	61.4
6410	24.5	solid	117°28′	82°47′			0.311	4.43	65.6
6410	33.1	liquid	116°13′	82°46′	−1°15′	−1′	0.304	4.36	64.9

temperature of 54°C to a value of 115°48′ at 73°C, while the corresponding values of 2ψ at these temperatures were 88°39′ and 88°40.8′ respectively.

It is doubtful however whether all of these changes in angles can be ascribed to changes in state. Small variations in phase difference, and still smaller variations in azimuth were observed for different temperatures, even when no change in state occurred. For example as shown in Table 2 for potassium mirror No. 9, a diminution in these angles was observed when the temperature of the mirror changed from 23.6 to 53.3 even though there is no change in state of potassium between these two temperatures. Of course a small change in the value of 2ψ for potassium will produce a correspondingly larger change in the coefficient of absorption k, since 2ψ is close to 90° and k enters as a function of the tangent of this angle.

Similar variations of phase differences were observed with change in temperature for liquid caesium. Using light of wave length 5890A, it was found that when the temperature was increased from 26.7 to 35.4°C, the phase difference was lowered by 1°37′. Increasing the temperature to 51° merely reduced the phase difference still further by 1°10′, the change being a gradual one. A repetition of the observations for wave length 5400A and for a temperature change from 30.2° to 38.2°C again revealed a lowering of the phase difference by 1°13′, there being no change in state.

In order to determine whether these small changes in phase differences and azimuth with temperature were really due to a change in the optical constants of the metals, or were caused by slight strains in the glass cells, an extended series of observations was made upon two empty glass cells, i.e., cells not containing any alkali metal. The

method of observation was the same as that on the metal mirrors, the light in this case being totally internally reflected from the inside surface of the glass window, i.e., the surface that is customarily in contact with the alkali metal. The changes in phase difference and in azimuth are listed in Table 4.

TABLE 4. *Empty glass cells.*

Cell No.	Wave length in A	Temp. °C	Observed changes in	
			Phase Δ	Azimuth 2Ψ
1	5400	23.8	−1°15′	
1	5400	34.6		
1	6410	21.0		
1	6410	33.5	−1°46′	
2	5890	23.5		
2	5890	65.0	−1°12′	−4′

The phase changes observed with variation of temperature for the empty glass cells are about the same as those found for the cells filled with the alkali metals. The change in twice the azimuthal angle for the empty glass cell No. 2 is about the same as that observed for caesium, but is not as great as that observed for potassium. It must therefore be concluded that the phase changes observed with the metals are due to a variation in the temperature of the glass cells. The cylindrical walls of the cells are made of pyrex glass, while the windows are of a good grade of crown glass. The windows were cemented to the pyrex glass, and it is very likely that with variation in temperature, there would be slight strains introduced to the difference in the expansions of the crown and pyrex glass. If there is any true variation in phase difference with temperature for the metals, it is so small as to be entirely masked by that of the glass cells. The azimuthal changes for caesium are inappreciable, but there is evidence of a slight diminution in azimuth for potassium with change in temperature.

Taking into consideration the various experimental results, it must therefore be concluded that within the limits of experimental errors, the liquid alkali surface is optically the same as the solid alkali surface.

DEPARTMENT OF PHYSICS,
CARNEGIE INSTITUTE OF TECHNOLOGY,
PITTSBURGH, PA., JULY, 17, 1928.

THE COLORATION OF KUNZITE AND HIDDENITE BY X-RAYS[1]

By P. L. Bayley

Spodumene, a natural lithium and aluminum silicate, sometimes occurs in large, clear crystals. The pale pink or purple variety from Pala, California, is called kunzite and the yellowish green variety from Alexander county, North Carolina, is called hiddenite. These varieties are also found in Madagascar and other places.

Doelter,[2] in his complete resumé of the effects of various radiations in coloring many chemicals and minerals, states that under Becquerel rays, kunzite takes on a somewhat bluish green hue while hiddenite, changing slightly, becomes about the same color. Much work on the luminescence and coloration of kunzite and other substances exposed to rays from radium preparations has been done by Przibram and his collaborators.[3] Nichols and Howes[4] studied the location of the absorption bands of unradiated kunzite and Coblentz[5] has measured the absorption in the infrared. Miss Bělař[6] has measured the absorption of unradiated and radiated kunzite at 445, 495, 570, 600, and 700 mμ as a function of the time of exposure to Becquerel rays. Wiedemann and Schmidt[7] made qualitative measurements of the absorption of radiated Na Cl, K Cl, and K Br, stating the position and width of the absorption bands in the visible region by use of the customary names of the spectral colors. It is peculiar that although the coloration of glasses, gem stones, and chemicals by Becquerel rays, x-rays, and cathode rays has been known for about 25 years, no spectrophotometric measurements

[1] P. L. Bayley. Phys. Rev., *29*, p. 353; 1927, Abstract 6; *31*, p. 1132, 1928, Abstract 95.

[2] C. Doelter. Das Radium und die Farben, Steinkopff, Dresden. 1910. pp 23-24, 74-75. Also Centralblatt f. Mineralogie, pp. 321-324; 1923.

[3] Karl Przibram. ZS. f. Phys., *20*, p. 196; 1923 (which includes a bibliography of previous papers). Phys. Zeit., *25*, pp. 640-643; 1924.

[4] E. L. Nichols and H. L. Howes. Phys. Rev., *4*, pp. 18-27; 1914.

[5] W. W. Coblentz. Supplementary Investigation of Infrared spectra. Part VI. Carnegie Institution of Wash. 1908. pp. 42-43.

[6] Maria Bělař. Akad. Wiss. Wien, Ber. 132, p. 45; 1923. But measurements of the spectral absorption of radiated kunzite were evidently used by Przibram in obtaining, for kunzite, the radio-photoluminescence per unit energy absorbed in the visible spectrum. Akad. Wiss. Wien. Ber. 131, pp. 511-530; 1922.

[7] Wiedemann and Schmidt. Annalen d. Phys., *64*, pp. 78–91, 1898.

of the colors were made until 1922. It seems that the need for quantitative study of such colors occurred independently to several workers at the Institut für Radiumforschung, Wien,[6] and the author.[8]

The following spectrophotometric study was made to see if the natural green of hiddenite is in any way similar to the green color produced in kunzite by x-rays.

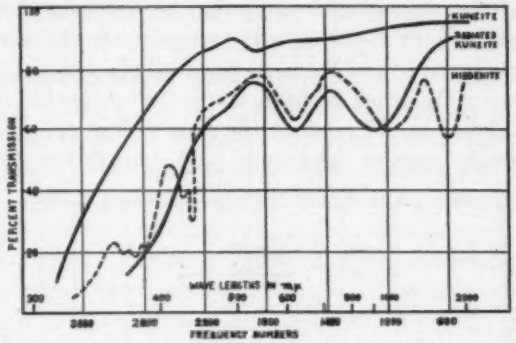

FIG. 1. *Showing the transmission of kunzite, radiated kunzite, and hiddenite from 300 to 2000 mμ.*

The sample of kunzite, from Pala, which was used in this study was 6.40 mm thick. Its color was pale pink. The two polished faces had a usable area, without a visible flaw, of about 5 x 20 mm. The sample of hiddenite from Alexander county was 4.1 mm thick. Although the polished faces were 20 x 35 mm, much of that area was opaque because of cleavage planes and occluded dark brown foreign material. An area of 4 x 17 mm was clear except for a number of dark brown spots which were just large enough to be visible upon close examination with the naked eye. Under magnification, the spots were seen to be transparent nebulous patches. Both samples were polished on the flat sides which customarily make the faces of the slabs of the material and which were approximately the clino-pinacoid faces.

The measurements in the ultraviolet portion of the spectrum were taken from plates exposed in a Hilger sectored disk spectrophotometer. For the visible portion of the spectrum, a Schmidt and Hensch Lummer-Brodhun spectrophotometer with contrast field was used. For the infrared region out to 2μ, a Hilger constant deviation spectroscope with glass lenses and prism and a Coblentz thermopile was used. The source of light was a concentrated filament tungsten lamp. For the region

[8] P. L. Bayley. Phys. Rev., *21*, p. 716; 1923. Abstract 52.

beyond 2μ, the Hilger constant deviation spectroscope with mirrors and rocksalt prism was used. A 220 volt Nernst glower was used as the source. In these latter two instruments, the current from the thermopile was measured with a high sensitivity Leeds and Northrup galvanometer.

The samples were placed in practically parallel beams of light, in all but the latter instrument, where they were mounted so as to be drawn in and out the beam as it converged from the mirror to the exit slit. This seemed to be the most practical place to put the sample because it was in the dispersed beam where the intense full radiation of the Nernst glower would produce the least fading action and where nearly the whole beam of light could pass through the small sample

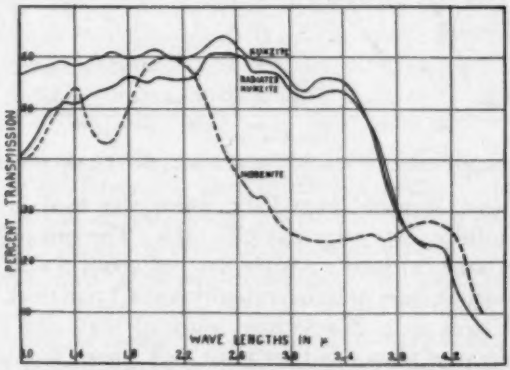

FIG. 2. *Showing the transmission of kunsite, radiated kunsite and hiddenite from 1.0 to 4.5μ.*

giving large deflections on the galvanometer. Thus the absolute values of the transmission as given in Fig. 2 can not be correct. But in measuring the transmission of natural minerals whose colors vary so much from sample to sample, as do the spodumenes, anything more than relative values are needless. Only the position and relative intensities of the absorption bands can have any significance.

The samples were measured in their original condition and again after 5 hours exposure to x-rays. They were placed within 5 inches of the target of a 30 ma aircooled, molybdenum target, Coolidge tube taking from 3 to 5 ma at 50,000 volts.

The results are shown in Figs. 1 and 2. The wave length scales are different on the two graphs so as to show best the shapes of the curves without compressing any portion too much. Both hiddenite

and kunzite transmit considerable radiation from 0.3μ to 4.5μ. The data for the region 1.2 to 2.0μ could not be measured accurately due to the small amount of energy which could be got through the glass system; hence in Fig. 1 the curves in that region are drawn smooth while the more accurate data in Fig. 2 show slight irregularities.

In the original kunzite, the absorption at 0.54μ accounts for the pale pink color. The hiddenite has a large number of bands, at 0.368, 0.378, 0.390, 0.432, 0.438, 0.630, 1.000, 1.670, 2.7, and 3.7μ.

After exposure to x-rays, the kunzite, quite bluish green, is seen to have developed absorption bands at 0.630 and 0.910μ. The shape of the curve differs enough from that of hiddenite to make the color appear slightly different to the eye. No trace of any absorption bands in the violet or ultraviolet could be found on any plate for the colored kunzite. Plates were made for short and long exposures with the sectored disk set for 20 and 40%, where the bands would show up the best, for freshly radiated samples. But if the bands existed either they were too faint to be found or they disappeared quickly under the light from the underwater spark before the spectrogram could be obtained on the plate. In a like manner, the sample was tested again and again to discover some trace of the band in the infrared at 1.670μ but no conclusive evidence could be got for its existence.

After the hiddenite was exposed to x-rays, no marked change was observed. The absorption in the visible was increased very slightly but no other change was found. The sample did not appear to change its hue to the more greenish blue of radiated kunzite as stated by Doelter.[1]

It is seen that x-radiation noticeably increases the absorption of the kunzite out to at least 3.5μ. From 1.0 to 4.5μ the two curves are very similar. Hiddenite has a much larger absorption than kunzite in the region from 2.2 to 3.8μ.

The coloration of kunzite is completely reproduceable. After twelve hours of exposure against the bulb of a 150-watt mazda lamp, the x-rayed kunzite returned to its original absorption at all wave lengths. Repeated coloration and fading did not produce any change in its transmission values.

Since there are monazite sands not far from the North Carolina hiddenite deposits, it is possible that in the period of formation of the deposits, radioactive materials were present and their radiations produced the green color.

COMPARISON WITH PREVIOUS WORK

Nichols' and Howe's[4] record for kunzite a broad band with its maximum absorption at 0.538μ. Through a 5 cm layer, bands were noted also at 0.4019, 0.4117, 0.4170, 0.4375, and 0.4501μ, and with a 1.5 mm layer a band at 0.380μ. These data might be expected to check with the author's data since the sample used was from Dr. Nichol's collection of kunzite and all the samples may have come from the same site. There is agreement in the main band at 0.54μ. It may be significant that the band noted between 0.3750 and 0.3875μ with its crest at 0.380μ and one at 0.4375μ correspond to the bands at 0.378 and 0.438μ in hiddenite.

The sample used by Coblentz[5] transmitted about 75% uniformly out to 4μ and then dropped almost linearly to 5% at 5μ. It was obviously more transparent in the region from 3 to 5μ than the author's sample. The apparent smoothness of Coblentz's curve may be due to the isolation of the points. Curves in Fig. 2 represent data taken at every 0.1μ and in many regions at every 0.05μ.

The sample of kunzite used by Miss Bělař[6] was also from Pala and when the data are plotted as spectral absorption curves at different stages of the coloration, the curve for the original sample has an absorption band at 0.54μ. As the process of coloration progressed, the absorption increased more rapidly in the red and violet so that the absorption became almost the same throughout the visible region, producing a grayish color. Further exposure caused coloration, still producing relative larger absorption in the red and violet with a maximum transmission at about 0.485μ. That is far from the author's value of 0.540μ. Although no data is given between 0.6 and 0.7μ, there is evidence of an absorption band between these limits although it is not nearly so pronounced as that found in the author's sample at 0.63μ.

This disagreement seems to cast some doubt on the possibility that such close agreement as obtained by the author can be true for all samples of kunzite. Tests on other samples from Pala and also from Madagascar should be made to see what agreement is possible.

A similar spectrophotometer investigation of the pink and green varieties of tourmaline might prove valuable. Doelter[8] and Hutchins[9] report that radiation turns the pink variety green.

[9] C. C. Hutchins. Phys. Rev., *27*, pp. 87–89; 1908.

The production by x-rays of an absorption band in the infrared (0.910μ) is so far unique. Przibram[10] found that gamma rays produced in calcite an absorption band in the ultraviolet, at 0.375μ. This may lead to the necessity of stating for samples apparently not colored by radiation that no *visible* coloration is produced. Possibly many substances which appear to be uncolored have bands produced in only invisible portions of the spectrum.

The author is indebted to the department of Physics of Cornell University for the use of its laboratory and for its facilities for two summers during which this investigation was made.

DEPARTMENT OF PHYSICS,
LEHIGH UNIVERSITY,
BETHLEHEM, PA.

[10] Karl Przibram. Phys. Zeit., *25*, pp. 640-643; 1924.

INSTRUMENT SECTION

AWARD OF PRIZE FOR 1927

Through the generosity of the Association of Apparatus Makers of the United States, the Journal of the Optical Society and Review of Scientific Instruments announced for the year 1927 a prize of two hundred and fifty dollars for the best paper on Scientific Instruments, other than optical, published in the Journal during that calendar year.

The committee for the award consisted of Professor C. E. Mendenhall, Chairman, University of Wisconsin, Professor A. G. Worthing, University of Pittsburgh, and Director L. O. Grondahl, Union Switch and Signal Company. After a very careful consideration of the large number of eligible papers, the Committee has unanimously agreed to award the prize to Professor E. L. Harrington, University of Saskatchewan, for his paper on A New Type of Aspirator of High Efficiency, appearing in the January 1927 issue, volume *14*, pages 87-97.

Honorable mention is given to the paper by Professor C. F. Wiebusch, University of Texas, entitled A Mechanical Harmonic Analyzer for Cathode Ray Oscillograms, and appearing in the December 1927 issue, *15*, pp. 355-8.

The editors desire to express their appreciation of the thorough and conscientious manner in which the committee after much study and consultation finally reached unanimous agreement.

The Scientific Apparatus Makers have offered similar prizes for the calendar years 1928 and 1929, and the Journal is deeply indebted to this organization for stimulating research in the fundamentals of Instrument Design.

The following summarizes the prize awards for the past three years.

1927 E. L. Harrington, A New Type of Aspirator of High Efficiency.
1926 Horatio B. Williams, The Einthoven String Galvanometer.
1925 Loyd A. Jones, New Method of Photographic Spectrophotometry.

[Editors]

A MOUNTING FOR THE PLANE GRATING

By George S. Monk

The principle objections to the use of a plane grating in the ordinary Littrow mounting are:

1. Absorption of the ultraviolet light by the glass lens. This is undoubtedly the most serious objection, as modern spectroscopic investigations are largely in the region of shorter wave lengths. It is also frequently necessary to get comparative exposures to lines in both the visible and ultraviolet regions. 2. Limitation to a comparatively small region of the spectrum at one time. As spectroscopic technique becomes more complicated, it is often necessary to make exceedingly long observations. If several settings are required to cover a region under investigation, much valuable time is lost, and in some cases critical conditions in the source are not easily duplicated. 3. Reflections from the surface of the lens. This can be avoided by tilting the lens slightly about a horizontal axis, thereby introducing a slight astigmatism, or else by the use of a thin opaque strip across the center of the lens, causing a loss of light. 4. Absence of automatic adjustment for focus. 5. Absence of normal dispersion. At a time when the principal work of spectroscopists was the identification and measurement of wave length, this was a serious objection. At present it is a minor difficulty for most work.

It is apparent that the deficiencies of the Littrow mounting are due to the use of a glass lens in the optical system. Several experimenters have substituted for the lens a mirror or system of mirrors, but in every case which has come to the writer's attention, the main features of the Littrow mounting other than the lens have been retained. The form of mounting described in this paper may have been used previously, but it seems to offer enough advantages for modern spectroscopic work to justify a detailed description.

From the theory of the concave grating is obtained the relation[1]

$$S = \frac{R\rho \cos^2 \theta}{\rho(\cos \theta + \cos i) - R \cos^2 i} \qquad (1)$$

[1] See Baly, "Spectroscopy;" Kaycer, "Handbuch der Spektroskopie;" or any well-known text on Physical Optics.

in which R is the radius of curvature of the grating, ρ is the distance from the slit to the grating, θ and i are the angles of diffraction and incidence respectively, and S is the distance from the grating to the spectrum lines in focus. For a plane grating, R is infinite, and in consequence

$$S = \frac{-\rho \cos^2 \theta}{\cos^2 i} . \tag{2}$$

If the light is convergent on the grating, as from a concave mirror, ρ will be the distance of a virtual source, and the sign is positive. If, also, i is constant, we obtain

$$S = A \cos^2 \theta \tag{3}$$

the equation of a lemniscate, in which $A = \rho/\cos^2 i$.

This use of a convergent beam from a concave mirror, and the location of the plane grating in the path of the beam, was suggested to the writer by Mr. Fred Pearson, chief optician at the Ryerson Laboratory, whose comments on the development of the experimental work in connection with this paper have been very valuable.

The general arrangement of the optical system is shown by Fig. 1, in which $i = 30°$. Light from the source is focused on the slit S by means of a suitable lens or mirror, thence passing to the concave mirror C.

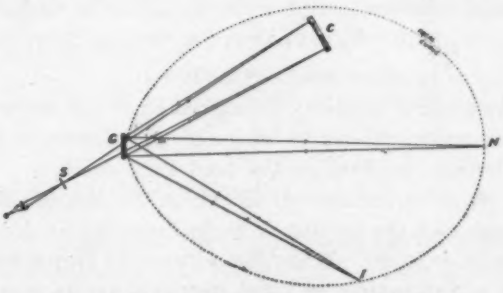

FIG. 1. *Optical Arrangement for Proposed Mounting*

From C, the light is reflected to a plane grating G. The spectra of successive orders on both sides of the central image I will be in focus[2]

[2] Cornu has shown that if there is a slight increase in the spacing of the rulings from one side of a concave grating to the other, the focus will not follow the true Rowland circle, but will deviate slightly from it. "Focal Anomalies," C. R., *116*, p. 1215; 1893.

on the curve represented by the dotted line. N is the position of the spectrum normal to the grating. For a fixed focal curve, i.e., a track on which the photographic plates can be clamped, the grating is not limited to one angle of incidence. In equation (3) the value of A determines the dimensions of the focal curve, and i may vary if ρ is made to vary inversely as $\cos^2 i$. In Fig. 2, O' is the position of the virtual source

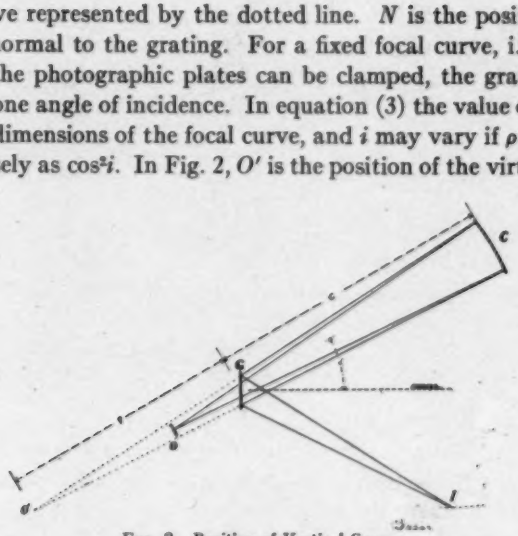

FIG. 2. *Position of Vertical Source*

at a distance ρ behind the grating, O is the slit source, and C, G, and I are mirror, grating and central image as in Fig 1. It will be seen that $\rho = GI$. Using for the concave mirror the simple relation $1/a + 1/b = 1/F$, $a = OC$ and $b = CG + GI$. We can impose the conditions that a, b, and F remain constant. But $b = c + \rho$. Also $\rho_2 = \rho_1(\cos^2 i_2/\cos^2 i_1)$. Hence $c_2 + \rho_2 = c_2 + \rho_1(\cos^2 i_2/\cos^2 i_1) = c_1 + \rho_1$ or

$$c_2 = c_1 + \rho_1(1 - \cos^2 i_2/\cos^2 i_1) \qquad (4)$$

where i_1 is the angle of incidence for a given setting of the spectrograph and i_2 is a new angle required to bring a given spectrum and order to a more advantageous position on the fixed focal curve.

Equation (4) gives the amount by which the distance between the concave mirror and the grating is to be increased or decreased as i is made smaller or larger. While the adjustment is not automatic, it can be made approximately to a high degree of accuracy, and a single focus plate should suffice for the exact determination[3]. In planning a mounting of this type it will be necessary to make the aperture of the mirror large enough for any value of i which may be necessary in order that the grating may always be filled. Perhaps the simplest mechanical

[3] Should the grating possess focal anomalies like those discussed by Cornu, the process of focusing will of course be more difficult and a fixed position for C will be better.

arrangement for changes in i and c would be a rigid bar connecting the slit and the concave mirror, capable of rotation about an axis, beneath the middle of the ruled surface of the grating and possessing a track of sufficient length to allow the mirror and the slit to be moved to any desired position along the bar. For stability, the ends of the bar could rest on bed plates. The grating would then be fixed at the end of the long axis of the focal curve. The slit could then be mounted at the end of a metal tube projecting beyond the principal enclosure of the spectrograph and fastened to the movable bar.

The most important condition is that the area of the grating be covered by the convergent light from C, which in turn should be large enough to intercept as large a cone of light from the slit as possible. It should be pointed out that the magnification of the spectrograph (for the central image) is b/a. Hence, for b greater than a the magnification is greater than unity with resulting loss of intensity for any portion of a spectrum line. In Fig. 1, the magnification is approximately five-thirds. If the slit is placed further away from the mirror the magnification becomes smaller, with resulting increase of intensity, provided that the angular aperture of the cone of light subtended by the concave mirror is made the same as before. While for object focus and image focus both finite the surface of a mirror should be an ellipsoid, for the apertures used a spherical mirror will be sufficiently accurate.

The spectrum is astigmatic. For the surface of a refracting medium whose index of refraction is n, the positions of the meridian and sagittal foci are given by

$$\frac{\cos^2 i}{\rho} - \frac{n\cos^2\theta}{s} = \frac{\cos i - n\cos\theta}{R}$$

$$\frac{1}{\rho} - \frac{n}{s'} = \frac{\cos i - n\cos\theta}{R}$$

where s' is the distance to the sagittal focus and the other symbols are the same as in equation (1). For the plane grating illuminated by convergent light we get the correct expressions by putting $R = \infty$ and $n = 1$, whence

$$\frac{\cos^2 i}{\rho} - \frac{\cos^2\theta}{s} = 0$$

$$\frac{1}{\rho} - \frac{1}{s'} = 0 \qquad (5)$$

The first of these is equation (3). From (5) is obtained the distance between the two foci for any value of θ and i:

$$s'-s=\rho(1-\cos^2\theta/\cos^2 i) \tag{6}$$

For $i=\pm\theta$, $s'-s=O$. Hence at the central image and at a position symmetrical with it on the other side of the lemniscate the astigmatism is zero. For a comparison spectrum for these portions of the focal curve an occulting bar or comparison prism can be used at the slit. From the second of equations (5) it is seen that the sagittal focus is a circle of radius ρ with its center at the midpoint of the grating. On the normal to the grating, $\theta=O$, and $s'-s=-\rho\tan^2 i$. Hence the sagittal focus of the slit source will be inside the focal curve. In order to bring a line perpendicular to the slit to a sharp focus on the fixed focal curve of the instrument, it will be necessary to increase b and decrease a. Thus for a comparison spectrum the occulting bar must be placed inside (behind) the slit. A comparison prism cannot well be used in this position because a second slit would then be required. Experimentally it has been found that with an opening in the occulting bar about one fourth as wide as the illuminated length of the slit, a position of the bar can be found for which the lengths of the comparison lines are short enough to be distinguishable from the lines of the other source over a considerable range of the focal curve on either side of the normal.

For a point image on the slit, the astigmatism will be given by $l(1-\cos^2\theta/\cos^2 i)$ where l is the length of the rulings of the grating. For $\theta=O$, the astigmatism will therefore be $-l\tan^2 i$ instead of $l\sin i\tan i$, the expression for the case of the Rowland mounting. For $\theta=\pm i$ the astigmatism will be zero, and for θ greater or less than $+i$ it will increase rapidly as θ increases. Kayser[4] gives for the astigmatism of the concave grating the expression $l(\sin^2 i/\cos i+\sin^2\theta)$. The following table shows a comparison of the astigmatism of the proposed mounting with that of the Rowland circle as utilized by Runge and Paschen in the circular mounting[5], and since installed in many spectroscopic laboratories.

The negative sign indicates that the sagittal focus lies inside the focal curve of the spectrum lines.

[4] Handbuch der Spektroskopie, Band I, 463.

[5] Anh. zu den Abhandlung d. Berlin. Akad. d. Wiss., 1902. Astrophysical Journal, *15*, p. 235; 1902.

TABLE 1. ($i=45°$)
Comparison of Astigmatism

	I Concave grating and circular mounting	II Plane grating and concave mirror mounting as proposed.
$\theta=$ 0°	$0.71l$	$-1.00l$
15°	$0.78l$	$-0.86l$
30°	$0.96l$	$-0.50l$
45°	$1.21l$	.0
60°	$1.46l$	$+0.50l$

If it is desired to photograph the region of the spectrum falling on C, the grating may be mounted with a slight forward tilt, or, better, the mirror C may be somewhat above a horizontal focal curve. This will add a negligible amount of astigmatism to that already present. Some difficulty is presented in the region of C owing to the possibility of direct illumination from the slit falling on the photographic plate, but in the experience of the writer this can be eliminated by proper screening.

The advantages of this type of mounting are:

1. The plane grating is made available for work in the ultraviolet. 2. The plane grating can be used to photograph simultaneously all wave lengths in several orders, an advantage hitherto possessed only by a concave grating combined with a circular mounting. 3. The use of a concave mirror (at C) of large aperture makes possible the use of a larger cone of light from the source. This is partly offset by the fact that the reflecting power of a metallic surface is never unity. The mirror may, however, be coated for special regions of the spectrum as often as necessary. In tests made of this type of mounting, a mirror of 445 cm focal length and 20 cm aperture, platinized by cathodic deposition, has been used with excellent results in the ultraviolet, when used in combination with a plane grating having a ruled surface about 11 cm by 4.5 cm. 4. A concave mirror is cheaper than a good lens of large aperture. 5. Reflections such as occur in the Littrow mounting from the surfaces of the lens are absent. There is a certain amount of light from the edges of the slit which will reach the plateholder in the region of the concave mirror, but this can be eliminated by proper diaphragming. 6. No extensive adjustments for focus and tilt are necessary, as in the Littrow mounting.

The disadvantages of the proposed mounting are:

1. The spectrum is astigmatic. But this feature is of recognized value in eliminating the effect of dust specks and unequal illumination on the slit. The astigmatism is greater than that of the Rowland mounting for positions on the focal curve normal to the grating. 2. The focal curve is not a circle. This prohibits the use of plateholders which can be clamped to any part of the focal curve. The plates themselves can be clamped to a properly shaped curve. The angle of incidence on the photographic plate at the flatter parts of the focal curve is somewhat larger than in the Paschen mounting. 3. Fewer orders can be used than in the Littrow mounting. This is also true of the concave grating in a Rowland mounting, but not of the Eagle mounting or any similar form. In fact, the chief advantage of the Littrow mounting is that it makes use of high orders with consequent high resolving power, and this is true whether the grating be concave or plane.

RYERSON PHYSICAL LABORATORY,
UNIVERSITY OF CHICAGO,
CHICAGO, ILLINOIS.
JUNE 24, 1928.

THE USE OF MIRRORS AS "EQUIVALENT SLITS" FOR X-RAYS

By H. E. Stauss

For many investigations using x-rays, fine slits are essential, as, for example, in x-ray spectroscopy or in experiments where approximately parallel radiation is desired. The construction of accurate slits, however, demands very nice machining. In addition, the maximum width of the slit, when used with x-rays, is uncertain. The metals that can be easily worked have comparatively small absorption coefficients and permit the x-rays to go through the edges if slits are made of them. The heavy metals machine so poorly that they cannot be used in accurate slits.

As yet no effort has been made to use reflectors as "equivalent slits," although their use seems to have several advantages. Their construction, if a suitable reflector is available, requires only rough machining. Very fine "equivalent slits" can be obtained easily and measured accurately. It has been found possible to obtain slit-widths of 0.012 mm with no trouble, and to determine the width with an error of approximately 0.002 mm. By the "equivalent slit" of a mirror is meant the slit, which, when placed in the proper position with respect to the image of the source as a source, transmits the same beam that the mirror reflects. The width "*s*" in Fig. 1 is the equivalent slit of the reflector. This width can be varied by merely turning the mirror, and can be measured in terms of the angle of reflection.

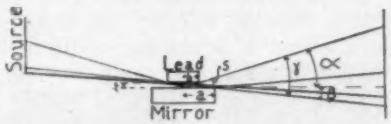

Fig. 1. *Diagram of a mirror used in place of a slit.*

In general, when x-rays are being reflected, it is possible to arrange the apparatus so that the mirror reflects only part of the original beam. The remainder goes past the mirror undeviated and may be used to determine the angle of reflection.[1] When a beam of x-rays that is being

[1] Cf. H. E. Stauss, Phys. Rev., *31*, p. 491; 1928.

reflected has a narrow angular width or when it has already passed through a narrow slit, there is no difficulty in separating the reflected beam from the one which is undeviated. Fig. 3 shows the separation of a beam that satisfies the second condition. The equivalent slit-width, "s," in such a case is the length of the mirror times the tangent of the angle of reflection, or $2a \tan \alpha$. To a close approximation α is one-half the angle between the extreme rays of the reflected and transmitted beams. Since α and γ are small

$$s = 2a\alpha = a\gamma$$

where "a" is the half-width of the mirror.

When a beam with a great angular width is being reflected, e.g., when the mirror is near the x-ray tube, there is some trouble in separating the reflected beam from the other since the two tend to overlap. For this case the following arrangement has worked very well. A gold-plated mirror, 1 cm wide and 3 cm long was faced by a lead strip 0.4 cm wide placed along its length. The two were separated at the ends by pieces of paper 0.03 mm thick. The combination was mounted, with the slit in a vertical position, on the head of the screw of a large binding-post as turntable. The mirror could be rotated about a vertical axis by a screw working against a wing projecting from the head. In the radiation that passed through the combination the reflected and the transmitted beams were well separated.

Fig. 1 shows the action of such a slit diagrammatically. The equivalent slit-width "s" is equal to $\tan \alpha$ times some length along the mirror which is determined by the origin of the extreme reflected ray. If the source is sufficiently broad, as it usually is, it is evident from the geometry of the figure that the extreme ray strikes the center of the mirror, and $s = a \tan \alpha$ where "a" is the half-width of the mirror. Since "α" cannot be determined accurately, while "γ" can be, "s" may be expressed in terms of "γ." Now

$$\gamma = \alpha + \beta$$

where "β" is the angle between the surface of the mirror prolonged and the extreme transmitted ray. Since "α" and "β" are small,

$$\alpha = \tan \alpha = \frac{t}{b}$$

$$\beta = \tan \beta = \frac{t}{a+b}$$

where "b" is the half-width of the lead and "t" the width of the slit between the mirror and lead. Therefore,

$$\frac{\alpha}{\beta}=\frac{a+b}{b}$$

$$\frac{\alpha}{\alpha+\beta}=\frac{a+b}{a+2b}$$

$$\alpha=\gamma\frac{a+b}{a+2b}$$

and

$$s=a\alpha=a\gamma\frac{a+b}{a+2b}$$

Errors in "a" and "b," due either to errors in measurement or to penetration of the x-rays through the edges, will have little influence on the determination of "s." The greatest error occurs in "γ," but a close upper limit can always be set for it, so that the maximum value of "s" is known, unlike the case of the ordinary slit.

Fig. 2 is a picture of the radiation transmitted by the slit described above at a distance of 120 cm. Here "γ" is somewhat less than 0.0033 radians, "a" is 5 mm, "b" is 2 mm. Consequently, "s" is less than 0.013 mm. Actually, it is more nearly 0.012. An error of 0.5 mm in measuring the separation of the two edges from which "γ" is determined would change the value by less than 0.002 mm. No effort was made to obtain smaller values of "s," but it would seem that, by using metal foil instead of paper to separate the mirror from the lead and by decreasing "a," "s" could be made considerably smaller. The line to the right in

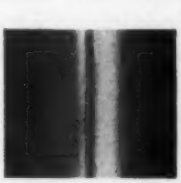

FIG. 2.

FIG. 3.

FIG. 2. *Radiation transmitted by a mirror used as a slit. The beam to the right is the one reflected by the mirror.*

FIG. 3. *Radiation reflected by a second mirror (line 2 mm to the right of the prominent unreflected beam). The magnification is two-fold.*

Fig. 3 shows the reflection of the beam by a second mirror, 3 cm long, when the distances between the two mirrors and between the second mirror and the film were 120 cm. It indicates what narrow beams can be obtained easily.

The adjustment of the reflector is very simple. When the eye is held close to the mirror, both the source and its image are visible. For a slit near the tube the object is the target. In practice the author has found that one edge of the target is more satisfactory than the focus for use in the adjustments. The mirror is rotated until object and image approach as near together as is desired. Extremely small displacements can be made in this manner even with rough instruments. It was found necessary, in the case of the slit described above, to leave one end of the mirror free in order to see the image. The actual determination of the width of the equivalent slit, of course, has to be made photographically with the x-rays.

The use of a reflector has a disadvantage in that a change in adjustment changes the direction of the beam and may throw it off the apparatus. However, the reflected beam is so broad, as Fig. 2 shows, that for small adjustments this is not a serious objection, because part of the beam is likely to remain in line. It should be remarked that, where two mirrors in series are being used to define a beam, the beam reflected by the second mirror may be slightly broader than is indicated by the geometry of the arrangement. This is due to the fact that x-rays are reflected somewhat diffusely. The diffuseness depends upon the nature of the reflecting surface and increases with the glancing angle of reflection; it may be reduced to a negligible minimum by the use of good surfaces and by setting the mirrors so as to reflect at small glancing angles. Where only a single narrow slit is desired, as in spectroscopy, the diffuseness is of no importance, since it does not appreciably influence the equivalent slit-width.

To counterbalance its disadvantages, the reflector has the advantages that it is easily constructed and that it is capable of producing very fine equivalent slits whose widths can be measured with a high degree of accuracy.

BRACE LABORATORY OF PHYSICS,
UNIVERSITY OF NEBRASKA,
JULY, 1928.

LOCUS OF SECONDARY IMAGE FOR TWO PLANE MIRRORS VARIABLE INDEPENDENTLY ABOUT PARALLEL AXES SITUATED IN THE RESPECTIVE PLANES*

By L. E. Dodd

ABSTRACT

The present problem, in geometrical optics, deals with a more general case of inclined mirrors than the simpler and familiar ones of the kaleidoscope, the "hinged mirrors," and the sextant. Here each of the two plane mirrors has its own independent axis in its own plane, the two axes being, however, parallel to each other, which requires that the intersection line of the two reflecting planes be always parallel to the two axes, but it may otherwise be located anywhere in space. The problem is stated so as to involve three parameters, in addition to the two independent variables. The main problem is that of the locus of the secondary image, O'', under these conditions. Derivations are given also for the locus of the primary image, O', and for the intersection, C, of the mirror planes. From the geometrical standpoint the problem is that of two related isosceles triangles, $O'AO$, $O'A_0O''$, where O is the object point and A, A_0, are the points where the mirror axes cut the principal plane, all five points being coplanar. The problem is made more general physically by supposing that the mirrors are half-silvered. The geometrical situation is expressed also in terms of intersecting circles, useful for graphical construction.

In devising an apparatus for certain psychological-optical tests, Dr. J. R. Liggett, of the Department of Psychology of this university, and the writer, found desirable the use of two plane mirrors that could be rotated independently of each other about separate parallel axes, one in each of the respective mirror planes. Further, it was necessary to know the precise position of the secondary image, rather than merely its directional location. An analysis of this problem, by the writer, is presented in this paper.

The optical situation is indicated in Fig. 1, where M_1 and M_2 are the perpendicular sections by the plane of the paper, of the two mirror planes. The respective axes are at A and A_0, and these two points are chosen to fix the x-axis, the point A_0 being taken as the origin. Then the respective angles ξ and η are the independent variables. Let $(\eta-\xi)=\epsilon$, the angle between the mirrors. If O is the point object it will have a primary image, O', in M_1, and a secondary image, in which this paper is particularly interested, O'', in M_2. An observer's eye in the vicinity of E will be able to see the image O'' by a narrow cone of rays represented by the ray shown in the diagram, Fig. 1. The three

* Paper presented at the meeting of the American Physical Society, Pacific Coast Section, at Pomona College, June 15, 1928.

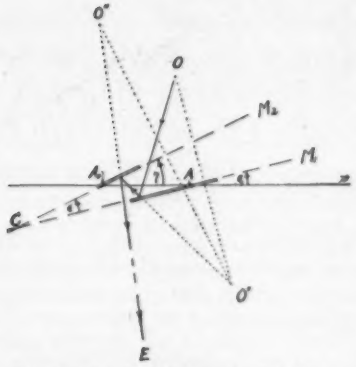

FIG. 1. *Physical situation of inclined mirrors in the present problem.*

points, A, A_0, O, chosen arbitrarily, require three parametric quantities. These may be taken to be the lengths A_0A, AO, A_0O,—all to be regarded throughout the present paper as positive. Let these lengths be denoted respectively by d, r, D. Or, particularly in solving the problem with rectangular coordinates, the three parameters may be taken as the length d and the two coordinates, x_1, y_1, of the point O.

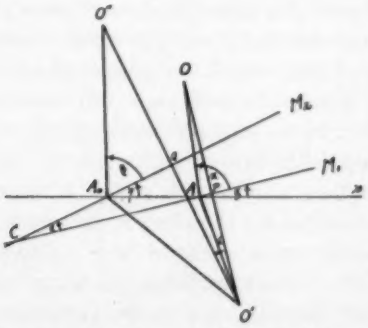

FIG. 2. *The geometrical problem of two related isosceles triangles.*

The problem can of course be stated and solved entirely geometrically, with no necessary reference to optics. As shown in Fig. 2, it is a problem of two related isosceles triangles, $O'AO$, and $O'A_0O''$, defined by the conditions of the problem, as follows: given, two respective apex points, A, A_0, each in a fixed position; the vertex point O of the first triangle is chosen arbitrarily, and the other vertex point, O', of the first triangle is always coincident with the corres-

ponding vertex point of the second.[1] Required, the locus of the remaining vertex point, O'', of the second triangle in terms of two independent variables and three parametric quantities.[2] The two independent variables may be taken as the angles ξ, η, which the two respective bisectors, M_1, M_2, of the apex angles make with the x-axis drawn through the two apex points, A, A_0, while the three parameters may be taken as already indicated above. Let the semi- angles, M_1AO, M_2A_0O'', be designated respectively by α and β.

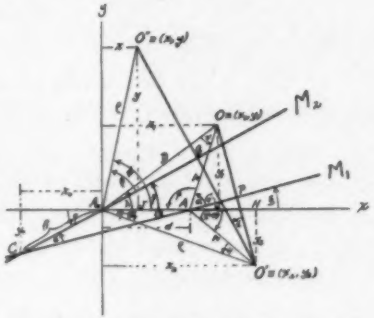

FIG. 3. *The geometrical problem in detail.*

In Fig. 3 this geometrical situation is presented in more detail. The angles AA_0O, OAA_0, A_0OA, of the parametric triangle AA_0O, are designated by λ, μ, ν, respectively, and angle $AO'A_0$ by ψ. Point C is the intersection of the plane of the diagram with the line of intersection of the two mirror planes. This intersection line is parallel to the two mirror axes. Denote the length of the side A_0O', of isosceles

[1] From this it is obvious that any series of isosceles triangles related to each other by a tip of one of the two equal vertices of the first Δ in the series being coincident with the tip of one of the two equal vertices of the Δ next in the series, and the tip of the other equal vertex of this second Δ being in similar contact with a tip of one of the equal vertices of the third Δ, and so on, and the apex of each Δ being fixed in position, can represent the principal section of a geometrical system of reflecting planes, each capable of independent rotation about a fixed axis in its own plane, all the axes being mutually parallel, the tips of the two equal vertices of any one of the Δ's representing point object and point image respectively for that mirror, the apex point being at its axis, and the image in that mirror becoming the object for the next mirror in the series. The principal section of the mirror itself is in each case the bisector of the apex angle.

[2] If the three points O, A, A_0, were to be located arbitrarily and perfectly generally in the xy plane, six parameters, the coordinates of the three points, would be required. Because the present restrictions place points A, A_0, on the x-axis and at a specified distance apart, d, three of the six possible parameters are removed.

$\Delta A_0 O'O''$, by ρ, the radius vector, and angle AA_0O'' by ϕ, the directional angle, of the secondary image point O''. It is somewhat simpler to use polar coordinates for the required locus of the secondary image. Moreover, it is convenient to retain them as functions of the rectangular coordinates, x_1, y_1, of the object point O.

The three internal angles, λ, μ, ν, of the parametric $\Delta A_0 AO$, are functions of the three parameters as follows:

$$\cos\lambda=(d^2+D^2-r^2)/(2Dd)\ ;\ \cos\mu=(r^2+d^2-D^2)/(2rd)\ ;\quad \cos\nu=(r^2+D^2-d^2)/(2rD)\ ; \tag{1}$$

or dropping perpendiculars OG, $O'H$, $O''J$, as in Fig. 3,

$$\cos\lambda=x_1/(x_1^2+y_1^2)^{1/2}\ ;\ \cos\mu=(d-x_1)/[(d-x_1)^2+y_1^2]^{1/2}\ ;\quad \cos\nu=[(x_1^2+y_1^2)-x_1d]/\{(x_1^2+y_1^2)\cdot[(d-x_1)^2+y_1^2]\}^{1/2}, \tag{2}$$

since $x_1=D\cos\lambda$, $y_1=D\sin\lambda$, $(x_1^2+y_1^2)=D^2$, and $[(d-x_1)^2+y_1^2]=r^2$.

The positions of four different points will be considered: (1) the object point, O; (2) the first image, O'; (3) the intersection, C, of the two mirror planes; and (4) the required second image, O''. However, the locus of O'' can be found analytically without knowing the position of the intersection C.

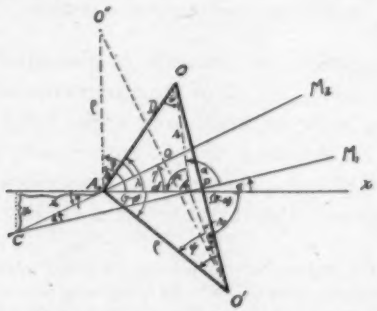

FIG. 4. *Geometrical situation, showing the intermediate triangle, $OO'A_0$, used in the derivations.*

DERIVATIONS

Polar coordinates. In polar coordinates the positions of the four points just mentioned are expressed by $O\equiv(D,\ \lambda)$, $O'\equiv[\rho,\ (\eta-\beta)]$, $C\equiv[\rho_1,\ (\pi+\eta)]$, and $O''\equiv(\rho,\ \phi)$. Of these four pairs of polar coordinates radius ρ is a function of the parameters and angle ξ alone, being independent of angle η. In addition to ρ, angle β is to be found so as to obtain an expression for the directional angle $\phi=\eta+\beta$, of point O'', and in order to find β use is made of angle ψ.

The expression for the first polar coordinate, ρ, of the secondary image O'', is obtained from $\Delta A_0 O' O$, which has been drawn with heavier lines in Fig. 4. Since $\overline{O'O} = 2\,\overline{OP} = 2\,r \sin\alpha$,

$$\rho^2 = D^2 + 4r \sin\alpha [r \sin\alpha + D \sin(\nu - \alpha)], \tag{3}$$

where $\alpha = (\lambda + \nu - \xi) = (\pi - \mu - \xi)$. The radius vector ρ serves both the first and the second images, O' and O''. β must now be found, and this angle can be used incidentally to find also the directional angle $(\eta - \beta)$ of point O'. From $\Delta A_0 O' A$, $\psi = (\alpha + \epsilon - \beta)$, so that $\beta = (\alpha + \epsilon - \psi)$, where $\cos\psi = (\rho^2 + r^2 - d^2)/(2\rho r)$, or $\sin\psi = [d \sin(\alpha - \xi)]/\rho$. Then

$$\eta - \beta = \eta - \epsilon - \alpha + \psi = \xi + \psi - \alpha \tag{4}$$

The radius vector ρ_1 of intersection point C is, from $\Delta A_0 C A$, $\rho_1 = (d \sin\xi)/\sin\epsilon$.

For the secondary image, O'', the directional angle then is

$$\phi = \eta + \beta = \eta + \epsilon + \alpha - \psi = 2\eta - \xi + \alpha - \psi. \tag{5}$$

Rectangular coordinates. In rectangular coordinates (see Fig. 3) the positions of the four points are expressed by $O \equiv (x_1, y_1)$, $O' \equiv (x_2, y_2)$, $C \equiv (x_0, y_0)$, and $O'' \equiv (x, y)$.

For the primary image, O', we have from $\Delta AO'H$,

$$x_2 = d + r\cos(\xi - \alpha), \text{ and } y_2 = r \sin(\xi - \alpha).$$

But

$$\cos\alpha = \cos[\pi - (\mu + \xi)] = -\cos(\mu + \xi) = (\sin\mu \sin\xi - \cos\mu \cos\xi)$$
$$= [y_1 \sin\xi - (d - x_1)\cos\xi]/[(d - x_1)^2 + y_1^2]^{1/2}, \tag{6}$$

and

$$\sin\alpha = [1 - \cos^2\alpha]^{1/2} = [y_1 \cos\xi + (d - x_1)\sin\xi]/[(d - x_1)^2 + y_1^2]^{1/2} \tag{7}$$

or,

$$\alpha = \pi - \mu - \xi = \pi - \xi - \cos^{-1}\{(d - x_1)/[(d - x_1)^2 + y_1^2]^{1/2}\}, \tag{8}$$

so that

$$x_2 = d + y_1 \sin 2\xi - (d - x_1)\cos 2\xi, \tag{9}$$

and

$$y_2 = -[y_1 \cos 2\xi + (d - x_1)\sin 2\xi]. \tag{10}$$

For the intersection C,

$$x_0 = \rho_1 \cos(\pi + \eta) = -\rho_1 \cos\eta, \tag{11}$$

and

$$y_0 = \rho_1 \sin(\pi + \eta) = -\rho_1 \sin\eta, \tag{12}$$

where $\rho_1 = d \sin\xi/\sin\epsilon$, as before. Or, since $y_0/x_0 = \tan\eta$, and $y_0/(x_0 - d) = \tan\xi$, then $x_0 = d \tan\xi/(\tan\xi - \tan\eta)$, and $y_0 = d \tan\xi \tan\eta/(\tan\xi - \tan\eta)$.

For the secondary image, O'', $x=\rho\cos\phi$, $y=\rho\sin\phi$.
But

$$\rho^2=x_2^2+y_2^2=(x_1^2+y_1^2)+2d[y_1\sin 2\xi+2(d-x_1)\sin^2\xi], \quad (13)$$

while $\phi=\eta+\beta$, (14)
where $\beta=\alpha+\epsilon-\psi$, and

$$\sin\psi=[d\sin(\alpha-\xi)]/\rho$$
$$=d[y_1\cos 2\xi+(d-x_1)\sin 2\xi]/[\rho[(d-x_1)^2+y_1^2]^{1/2}] \quad (15)$$

CHECKING THE EQUATIONS

The expressions given above for the secondary image were checked for different cases as follows: (1) $\xi=\pi/2$; (2) $\xi=0$; (3) $\eta=\pi/2$; (4) $\eta=0$; (5) $\xi=60°$; (6) $\xi=\eta$. The locus was plotted for each case,

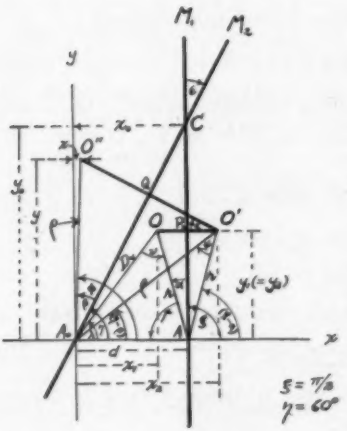

FIG. 5. *Particular case:* $\xi=\pi/2$; $\eta=60°$ *approximately.*

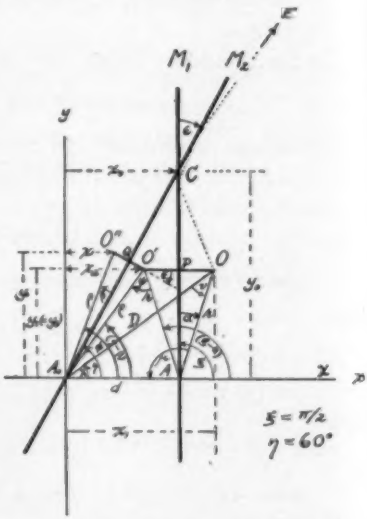

FIG. 6. *Particular case:* $\xi=\pi/2$; $\eta=60°$.

with a total of five calculated points on each curve, for definite values of the independent variable, -0, 20, 45, 60, 90°. Moreover, graphical construction in detail was performed for six particular and corresponding cases: (1) $\eta=45°$; (2) $\eta=20°$; (3) $\xi=60°$; (4) $\xi=90°$; (5) $\nu=0°$; (6) $\xi=45°$;—in order to check calculated values of α, β, ψ, ρ_1, x_2, y_2, x_0, y_0. The graphs and constructions mentioned above are being omitted from this paper. Constructions are given in Figs. 5, 6, 7, 8, for additional cases as indicated on the diagrams themselves.

Fig. 6 presents a special case that is geometrical only, provided that the mirrors are opaque and reflecting on their opposed faces, as suggested in Fig. 1. But even with M_1 opaque, this case is possible optically if this mirror is silvered on the back also, so that the image O'' could be seen by an eye at E, by means of a cone of rays from the object as indicated by the short-dashed line. But it appears that cases possible in the geometrical sense only, with opaque mirrors, may be possible

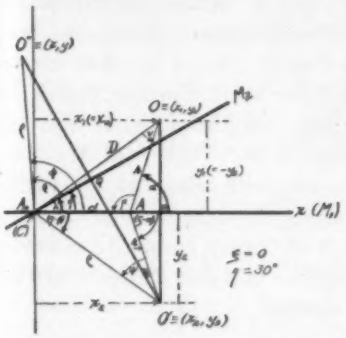

FIG. 7. *Particular case:* $\xi=0$; $\eta=30°$.

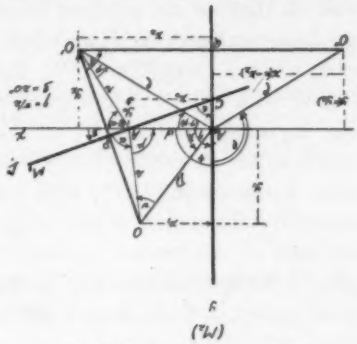

FIG. 8. *Particular case:* $\eta=\pi/2$; $\xi=20°$.

optically also if the mirrors are half-silvered. Mirror planes regarded as half-silvered transmit rays as well as reflect them from both faces, and in this way the optical situation has become more general.

LIMITS OF VISIBILITY OF SECONDARY IMAGE

In the present geometrical derivations the mirror planes have been assumed of indefinite extent. Whether the secondary image is visible to an eye in a particular position, will depend upon where the edges of the reflectors are located, in other words it will depend upon the width and the locations of the physical mirrors. Starting with reflecting planes of infinite extent, we may imagine them cut away suitably for visibility of the secondary image for a particular experimental case. Thus Fig. 1 indicates two physical mirrors giving visibility of O'' to an eye at E, for the particular distance between mirror axes, values of ξ and η, and direction of view of the observer.[3] But in Fig. 1 the two mirrors might be of infinite extent and still give visibility of O'' to an

[3] The number of images visible in the ordinary hinged mirrors, where the surfaces are regarded as reflecting clear to the vertex, is treated in a recent paper by H. Maurer, Physikalische Zs., *29*, No. 5, pp. 147–149; March 1, 1928. The author cites two other papers on the subject. (See Science Abstracts, 1928, abstract NO. 1540, also NO. 1229.)

eye at E, if they (particularly M_1 in this case) are half-silvered, as suggested above. Two or more half-silvered mirrors could in a sense be referred to as "generalized mirrors," since they may afford a visibility of images impossible with opaque mirrors under the same conditions.

GRAPHICAL CONSTRUCTION

In Fig. 9 it is seen that the position of the secondary image O'', as well as that of the primary image O', can be located graphically by the intersection of any two of three different circles, whose respective radii are r, ρ, and $R(=\overline{CO''})$. Points O and O' lie on the same circle, $O'geO$, with known radius r, and with center at A. Further, this circle has its image circle, efO'', in mirror M_2. This image circle also has radius r, but its center is at A', the image in M_2 of the point A. Also, there is the circle $O'hO''$, with radius ρ and with center at A_0. Circle $O'kOO''$, with radius R and with center at C, is the familiar circle in the case of the hinged mirrors,—where its center is on the common axis of the mirrors (the axis of the hinges),—which is the locus of the point object and the double series of images.

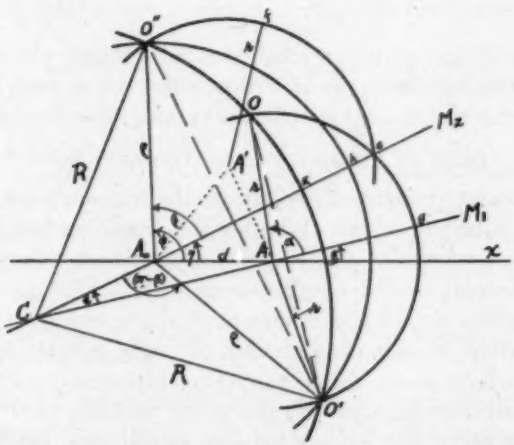

FIG. 9. *The geometrical situation in terms of intersecting circles.*

In a graphical construction to locate O'', radius R is readily taken equal to $\overline{CO}$, and with C as center an extended arc $O'kOO''$ is drawn. Also, with A as center and r as radius, arc $O'geO$ may be drawn. One intersection of these two arcs is the position of the primary

image O'. $\overline{A_0O'}$ is then ρ, the radius for arc $O'hO''$, with A_0 as center, thus locating O'', so that arc efO'' can be omitted, which is desirable as its construction requires the locating of A'.

[It may be noted that the analytical expression for R, obtained from $\Delta O''A_0C$ (see also Fig. 3), is

$$R^2=\rho^2+\rho_1^2-2\rho\rho_1\cos\left[\phi-(\pi+\eta)\right], \tag{16}$$

or, rewriting,

$$R^2=\rho^2+(d\sin\xi)^2/\sin^2\epsilon+2\rho d\cos(\phi-\eta)\sin\xi/\sin\epsilon. \tag{17}$$

In these two equations R is a function of ϕ and ρ. A simpler expression is obtained from ΔOA_0C, $\overline{CO}^2=\rho_1^2+D^2-2\rho_1D\cos\sqrt{OA_0C}$, where $\sqrt{OA_0C}=\pi-\sqrt{QA_0O}=\pi-(\lambda-\eta)$, so that $\cos\sqrt{OA_0C}=-\cos(\lambda-\eta)$. Then

$$R^2=\overline{CO}^2=\rho_1^2+D^2+2\rho_1D\cos(\lambda-\eta). \tag{18}$$

When written in terms of independent variables and parameters, (18) becomes

$$R^2=\left\{D^2+(d\sin\xi/\sin\epsilon)^2+\left[(2Dd\sin\xi)/\sin\epsilon\right]\cdot\left[\left(\frac{D^2+d^2-r^2}{2Dd}\right)\cos\eta+\sin\cos^{-1}\left(\frac{D^2+d^2-r^2}{2Dd}\right)\sin\eta\right]\right\}. \tag{19}$$

It is perhaps not advantageous to make use of these analytical expressions for R.]

Briefly, given the points O, A_0, A, C, point O'' may be rapidly located graphically by constructing one arc each of three circles, say in the following order: (1) $O'geO$; (2) $O'kOO''$; (3) $O'hO''$,—with respective centers at A, C, and A_0, and respective radii r, $R=\overline{CO'}$ and $\rho=\overline{A_0O'}$. If the two mirrors are near parallelism, C may be too far removed from A_0 and A for use in the graphical construction. But O'' is easily located for the case of strict parallelism, and two approximately minimum (as to practicability) values of ϵ, positive and negative respectively, can be used as points on the plotted locus nearest the point for parallelism, one on each side of that point.

A different graphical procedure is as follows: construct arc $O'gO$ (Fig. 9), center at A, radius r; draw $OO'\perp M_1$, intersecting M_1 at O'; construct arc $O'hO''$, center at A_0, radius A_0O' $(=\rho)$; draw $O'O''\perp M_2$, intersecting M_2 at O''.

CONCLUSION

The present paper, in geometrical optics, deals with a problem of inclined plane mirrors that is somewhat more complex than certain well-known cases, viz.; (1) inclined mirrors at a fixed angle, as in the kaleidoscope, where the angle is constant usually at 60°; (2) the demonstration device of two hinged mirrors, with the angular separation variable, but with a common axis; (3) the familiar sextant, where one of the mirrors, the horizon glass, is stationary, while the other mirror, the index glass, is rotated about an axis in its own plane and parallel to the plane of the fixed mirror,—in which instrument the line of intersection of the two mirror planes is variable in position, but is located always in the stationary plane of the horizon glass. The present situation is somewhat more complex in that both mirrors are movable independently, each about its own axis in its own plane, the two axes being parallel to each other, and consequently the line of intersection of the mirror planes, while being always parallel to the two axes, is free otherwise to move to any position in space, determined by the mutually independent inclinations of the two mirrors with a reference line drawn perpendicularly through both axes. Equations have been derived for the locus of the secondary image under these conditions, and incidentally, corresponding expressions for the primary image, and for the intersection of the two mirrors.

DEPARTMENT OF PHYSICS,
UNIVERSITY OF CALIFORNIA AT LOS ANGELES.

A USEFUL DESIGN OF TUBE-VOLTMETER

By W. F. Powers and G. W. Alderman

The design of tube-voltmeter described below will be seen from Fig. 1 to be an incorporation into a handy assemblage of the types of voltmeter of Van Der Bijl[1], of Moullin[2], of Kirke[3], and of the balanced-galvanometer principle of Dowling[4]. Thus this unit is very convenient in a physics laboratory. For the theory and method of use, particularly in very low frequency and in radio frequency measurements, the reader is referred to the work of Van Der Bijl and of Moullin.

Fig. 1 gives the circuit. The black dots represent binding-posts. A, B, C_1, C_2 and B_1 are batteries; V_1 and V are dc voltmeters; G is a portable or wall-type galvanometer; M is a microammeter or sensitive galvanometer; R_1 is a variable resistance box; C is a one micro-farad

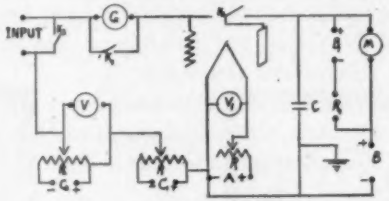

Fig. 1. *Schematic wiring diagram.*

condenser; K_1 and K_2 are switches or tap-keys; K_3 is a s.p.d.t. switch; P_1 and P_2 are high resistance circular potential dividers. A metal panel and base are used. On the panel are mounted V_1, P_1 and P_2, filament rheostat R, and binding-posts to which V, G, M and R_1 may be connected. At the rear of the base are the binding-posts for the various batteries. The input terminals are on the left side of the base.

It may be noted that with K_1 open and K_2 closed to the left there results the "slide-back" type of tube-voltmeter of Kirke (loc. cit.). We have introduced potential divider P_1 and dry cell C_1 as the means of adjusting the reference value of the current through G to zero or to a

[1] Van Der Bijl, Thermionic Vacuum Tube, first edition, p. 368.
[2] Moullin, Radio Frequency Measurements, p. 35; 1926.
[3] H. L. Kirke, Experimental Wireless and The Wireless Engineer, pp. 67–76; Feb. 1927.
[4] J. J. Dowling, Phil. Mag. pp. 81–101; July 1923.

predetermined small value when the input is shorted by closing K_3, and when P_2 is set for zero reading of V. When making readings of dc and ac peak voltages the "slide-back" value of voltage is thus given directly by V. This arrangement of connections provides a convenient indicator for a wavemeter circuit.

With K_2 closed to the right, B and M connected, there results a tube-voltmeter of the Van Der Bijl type (loc. cit.). By using in addition, B_1 and R_1 the plate current through M may be brought to any convenient value or balanced out for any condition of input voltage. Without this balancing battery branch this scheme of connections may be used for the determination of some static characteristics of vacuum tubes.

When K_2 is closed to the right, the B binding-posts connected together, M inserted, (B_1 and R_1 not used), a circuit results similar to that of the Moullin thermionic voltmeter.

We have found that this foundation unit with associated apparatus is very frequently useful in connection with dc and ac measurements; for example in the verification of Kirchhoff's Laws for such a dc scheme as suggested by M. N. States[5] and in obtaining data for vector diagrams in ac circuits of commercial frequency.

The chief advantages of this foundation unit are that it does not permanently tie up costly pieces of apparatus, that it may be widely used in electrical measurements, and that it is stimulating to students.

MASS. AGRIC. COLLEGE,
AMHERST, MASS.,
JUNE 1928.

[5] M. N. States, J.O.S.A. & R.S.I., *14*, pp. 328–331; 1927.

A DEVICE FOR ACCURATE TIMING*

By Herrick L. Johnston

In carrying out very accurate calorimetric measurements in progress in this laboratory, in which energy is supplied electrically and computed from the usual factors of voltage, current and time, it appeared desirable to obtain higher accuracy in the time factor than can be gained by the use of a stopwatch and hand control switches. Through the courtesy of Professor R. T. Crawford and of Professor C. D. Shane of the Astronomy Department of this university, accurate time signals were available for our use, and it remained to find a practicable method for utilizing these signals to control the timing electrically. White[1] describes a switch for whose original form he gives credit to Dr. Dorsey of the United States Bureau of Standards. With this switch as a starting point, we have developed the device illustrated in the accompanying diagram. This device has proved mechanically reliable, is simple, is compact and, in our experience, has proved to give an accuracy of

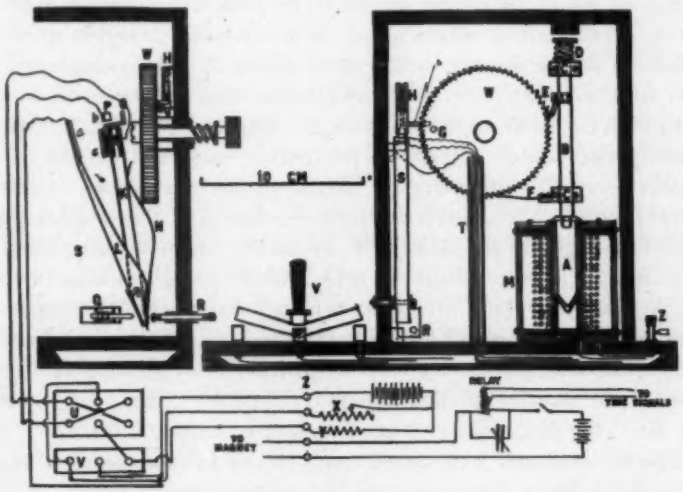

Diagram of timing device

* Contribution from the Chemical Laboratory of the University of California.

[1] W. P. White, Phys. Rev., *31*, p. 688; 1910.

greater than 3×10^{-3} seconds, on any measured interval, which is tenfold beyond our requirement. In our measurements we allow the energy batteries to discharge prior to the heating period through a stabilizing resistance about equal to that of the calorimeter heater. Our requirements are, therefore, to change the current from the stabilizing resistance to heater at the beginning of a determination, to supply energy for an integral number of minutes and to change the current from heater back to stabilizing resistance at the end of the period. The wiring can be modified, of course, to meet other requirements.

The magnet *M*, operated on a current of about four amperes and energized through a relay by astronomical time signals received at second intervals, consists of a thin brass tube fastened at either end to bakelite squares, wrapped with about 500 turns of asbestos insulated No. 22 copper wire, and the whole surrounded by two or three layers of thin soft iron. A larger number of turns would probably be superior as this would give the same force with a smaller current through relay contacts. The soft iron armature *A* is shaped at the tip to give a conical air gap and is grooved once vertically (not shown in the drawing) to permit escape of air during stroke. The armature is attached to the square brass shaft *B* which moves in bearings *C*, *C* and is normally raised out of the magnet by the stout spring *D*. The amplitude of *B* is so adjusted that the pawl *E* rotates the sprocket wheel *W* by one tooth with each downward stroke of the magnet. The flat steel spring *F* serves the double purpose of preventing backward motion of the sprocket and of overcoming the inertia of the moving wheel during forward motion. With each complete revolution of the wheel, the pin *G*, at right angles to the plane of *W*, opens the sensitive spring latch *H*. *H* is simply a slide bolt held down by a weak spring and raised by the attached pin "*h*." This latch is so adjusted that the pin *G* engages the latch lightly at one stroke of the magnet and raises it at the full movement of the next stroke. By making the number of sprocket teeth just equal to the number of time signals received per minute (in our case fifty-nine) the latch is thus opened at exactly minute intervals.

The reversal switch *S*, shown more clearly in the side view to the left of the diagram, is modeled after the form designed by Dorsey and White as previously mentioned, and possesses the advantage that the electrical circuits which it controls are both opened and closed by almost identical mechanism, and thus the time error of mechanical lag is practically cancelled out of the timed intervals. *K* is a square bar of

aluminum pivoted in the arm L which is constructed of thick sheet bronze bent into the proper shape.* K and L are normally pressed together by the strong steel spring N and an electrical circuit thus closed through the flat contacts "a" and "c." The aluminum arm is insulated from the contact "c" and its lead wire by bakelite strips and a bakelite tube shown in black in the diagram. When the switch is "set" by pressing the push button R and thus engaging the brass strip O in the latch H, the arm L is stopped by the square binding post P, thus closing the circuit "ab" and putting "c" on open circuit. When the latch H is raised and O is thus released the pressure of the spring N drives the contact "c" against "a," at the same moment opening the circuit through "b." Q is a spring plunger which prevents L from rebounding to P. Platinum was used for the contacts "a," "b" and "c" since previous experience with copper contacts showed that corrosive metals were unreliable due to varying contact resistance. The lead wires from these contacts were silk and rubber insulated flexible copper cable and were carried through the bakelite tube T to the false bottom of the instrument and there distributed to respective switches and terminals. The entire instrument was encased in bakelite and provided with a glass front.

The wiring plan is shown at the foot of the diagram. U is a double pole, double throw switch by means of which the reversing switch when set may be made to complete the circuit through either the stabilizing resistance, represented at X, or the heater circuit, represented at Y. Tripping the reversing switch by raising H reverses this order. V is a single pole, double throw switch shunted across the circuits controlled by S and U. Thus in the minute prior to commencing a determination the reversing switch is set to change the current from stabilizing resistance to heater, and V is put on open circuit. After the latch H is tripped V is closed to the heater and U is opened. In the final minute preceding the close of the measurement the reversing switch is set to change the current from heater back to stabilizing resistance, and V is opened.

It is of course desirable that the resistance be the same through the various energy circuits employed in the device, or, at least, that differences by different circuits be an inconsiderable portion of the entire heater circuit resistance. We have measured the resistances at the binding posts leading to Y and find for the circuit through V a resistance of 0.039 ohm; for the circuit through U and "ab" a resistance of

0.042 ohm; and through U and "ac" a resistance of 0.039 ohm. Frequent tests of the resistances between the platinum contacts have also shown the contact resistances constant to 0.001 ohm. During the five or ten seconds at the start and at the end of the heating periods when U and V are both closed while changing from one to the other the resistance between terminals is only 0.024 ohm. Considering the length of our heating intervals and the resistances employed in our heaters, these differences are entirely negligible in comparison with other factors in our calorimetry.

For the calibration of this instrument we are indebted to Dr. R. E. Cornish who carried out the calibration by comparing a four minute time interval controlled by the timing device with the number of oscillations of a high frequency oscillating circuit of constant frequency, by the method used by Cornish and Eastman.[2] The results obtained in five comparisons are as follows:

Calibration results

Run	Number of oscillations
1	106622
2	106623
3	106624
4	106624
5	106621
Average	106623

The mean deviation from the average is one oscillation, which corresponds to 2.3×10^{-3} seconds, and the extreme variation is three oscillations, which corresponds to 7×10^{-3} seconds. Since the experimental error of this method of calibration is stated by Dr. Cornish to be of the order of one full oscillation of the circuit it is evident that these variations are hardly beyond the experimental error and the accuracy of timing is probably in excess of the value $\pm 3 \times 10^{-3}$ seconds. It is obvious that, within the limits of accuracy of the time signals themselves, this is an error in the absolute sense since it is the cumulative error of one operation of starting and one operation of stopping the current through the heater circuit.

[2] Cornish and Eastman, J. Am. Chem. Soc., *50*, p. 627; 1928.

One or two practical observations affecting the attainment of this accuracy should perhaps be stated. In order to avoid appreciable variations in the effect of mechanical lag it is desirable that the action of the magnet be both powerful and rapid and that the spring N be as stiff as possible without causing undue difficulty in setting the switch. Another possible source of annoyance is the occasional sticking of a relay contact in the circuit leading to the time signals with the subsequent "loss" of a second in the timing. Obviously this can largely be avoided through proper attention to the relays, but, by a simple expedient, we guard against the possibility of such occasional error. Time signals are received at second intervals for fifty-nine seconds and then a pause occurs for two seconds. By so adjusting the phase of the sprocket that the latch H trips on the signal immediately following the pause it is easily possible to detect the "loss" of an occasional second by noting the relative interval between the pause and the trip. Should such an occurrence happen during a measurement we can thus apply the proper correction which must be an integral second.

In conclusion we wish to express our thanks to Professor W. F. Giauque for his interest in this work and to Mr. G. F. Nelson and assistants for aid in the construction and mechanical design of the instrument.

SUMMARY

1. A device has been described for securing accurate timing in electrical circuits.

2. The instrument has been calibrated and shown to possess an accuracy of better than 3×10^{-3} seconds for any integral number of minutes.

UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA.

ROTATIONAL INERTIA APPARATUS WITH SPARK RECORDING

By Herman E. Seemann

The method which was in use in one of the undergraduate laboratories here for obtaining the record in the rotational inertia experiment made use of a stylus driven by an electromagnet. The stylus kicked when an electric circuit was closed by a clock pendulum and mercury contact. The record was taken on glazed paper fastened to the edge of the disc and smoked with a candle flame. Before measuring the record the student "fixed" it in dilute shellac. The fact that a large number of students performed the experiment made the use of soot and shellac a decided nuisance and did not encourage neatness.

After some experimenting with other arrangements it was decided to adapt the spark method of recording to the apparatus then in use. This method has already been described*. It was only necessary to fasten a hard rubber block to the movable arm which originally held the electromagnetic stylus. (See photograph.) A flexible insulated

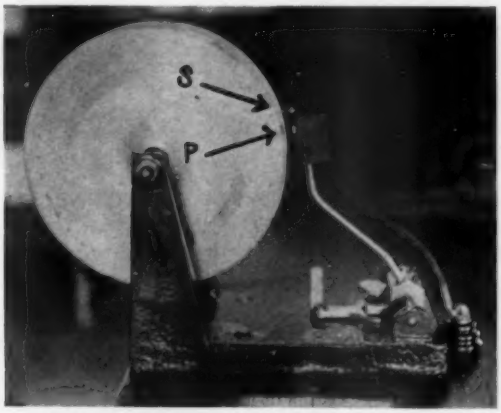

Fig. 1. *Rotational inertia apparatus with spark recording device.*

* Behr and Reynolds, J.O.S.A. & R.S.I., *13*, p. 213; Aug. 1926.

wire from the high potential was connected to a small pointed machine screw "P" countersunk in the block. A two to three millimeter spark gap was satisfactory. An insulated stylus "S" which made a fine scratch on the paraffined paper facilitated the assignment of the spark holes to their respective revolutions of the disc. Four instruments were connected in parallel and their bases grounded so that the chance of shock was small. Only one record could be taken at a time but time was saved in the preliminary operations.

The high potential source already in use on our free fall apparatus could have been used on this apparatus except for conflicts in the schedule of experiments. The unit built was similar except that an 1800 rpm synchronous motor was geared down 15 to 1 to a commutator supplied with 200 volts D.C., thus giving a half second interval between sparks. A revolution counter was permanently connected to the commutator shaft to facilitate the student's determination of the speed.

DEPARTMENT OF PHYSICS,
CORNELL UNIVERSITY,
ITHACA, N. Y.

Introduction to Modern Physics. By F. K. Richtmyer. vii+596 pages. McGraw-Hill Book Company, Inc., New York. $5.00.

Professor Richtmyer is to be congratulated on writing an interesting book which fills a real need in the teaching of physics in American colleges. This treatise is of a grade of difficulty such that it may profitably be used by a student who has had elementary courses in light, mechanics, and electricity and magnetism.

The first seventy-five pages are taken up by an historical sketch beginning with the Greeks, which is interesting, and will give the reader valuable perspective, but which cannot be classed as modern physics. The reviewer feels that this portion of the book might well be shortened to make room for more recent developments. Next, the electromagnetic theory of light is discussed with an admirable sense of the topics which are really important. A chapter is devoted to the elucidation of the field of a moving charge. In these chapters every attempt is made to make the mathematical derivations as clear as possible. The physical significance of each question is held in the foreground. Next, we encounter three chapters on the photoelectric effect, the origin of the quantum theory and specific heats. The remainder of the book deals with atomic structure from the standpoint of Bohr's theory. Attention may be especially directed to the chapters on x-rays and on nuclear physics. In the chapters on atomic structure physical considerations predominate, and no attempt is made to introduce material which will appeal to the research worker in atomic structure. It is perhaps to be regretted that Professor Richtmyer has not included at least the elementary concepts of wave mechanics, since there is no reason why these matters should not be presented to readers of the type for whom this book is intended.

Almost all of us have had the experience of being approached by some excellent student, who has completed a year or two of physics, with the question, "What book can I read to give me a general oversight of the more modern parts of physics?" Professor Richtmyer's volume may be recommended as a very adequate answer to this question.

ARTHUR EDWARD RUARK